

Definitions

In statistics, a **hypothesis** is a claim or statement about a property of a population.

A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.

Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

Key Concepts

The role of the following should be understood in order to understand hypothesis testing:

- ❖ null hypothesis
- ❖ alternative hypothesis
- ❖ test statistic
- ❖ critical region
- ❖ significance level
- ❖ critical value
- ❖ P -value
- ❖ Type I and II error

Objectives

- ❖ **Given a claim, identify the null hypothesis and the alternative hypothesis, and express them both in symbolic form.**
- ❖ **Given a claim and sample data, calculate the value of the test statistic.**
- ❖ **Given a significance level, identify the critical value(s).**
- ❖ **Given a value of the test statistic, identify the P -value.**
- ❖ **State the conclusion of a hypothesis test in simple, non-technical terms.**

Components of a Formal Hypothesis Test

Null Hypothesis:

$$H_0$$

- ❖ The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is **equal to** some claimed value.
- ❖ We test the null hypothesis directly.
- ❖ Either reject H_0 or fail to reject H_0 .

Alternative Hypothesis:

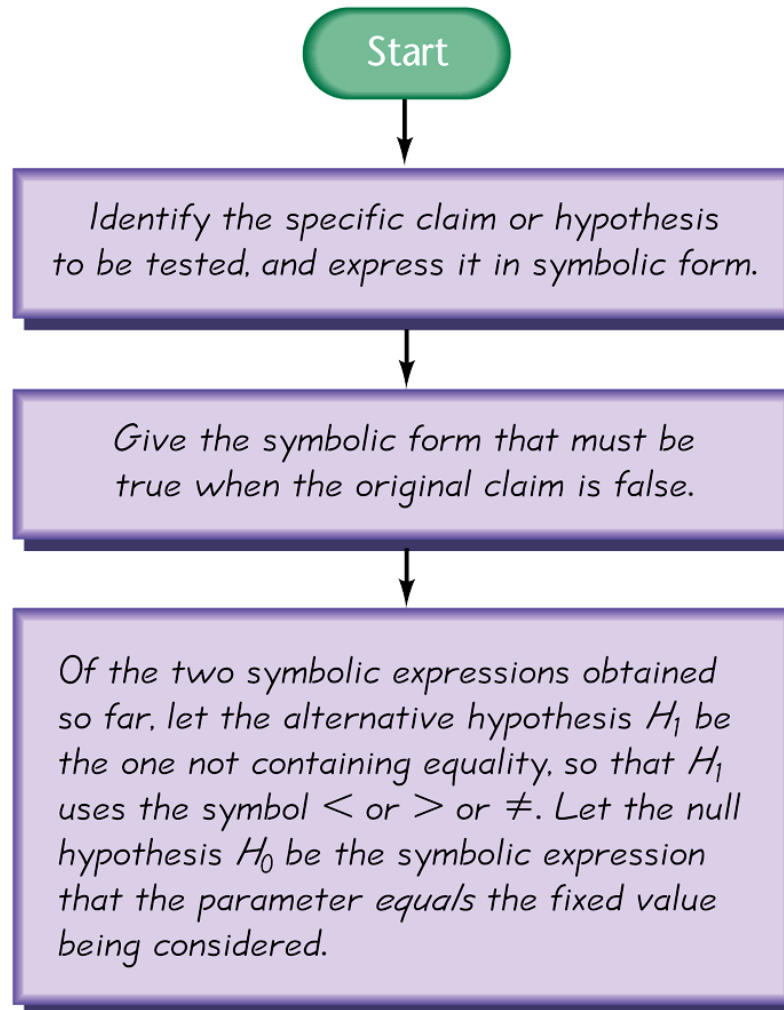
$$H_1$$

- ❖ The **alternative hypothesis** (denoted by H_1 or H_a or H_A) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- ❖ The symbolic form of the alternative hypothesis must use one of these symbols: \neq , $<$, $>$.

Note about Forming Your Own Claims (Hypotheses)

If you are conducting a study and want to use a hypothesis test to **support** your claim, the claim must be worded so that it becomes the alternative hypothesis.

Note about Identifying H_0 and H_1



Test Statistic

The **test statistic** is a value used in making a decision about the null hypothesis, and is found by converting the sample statistic to a score with the assumption that the null hypothesis is true.

Test Statistic - Formulas

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for proportions

$$z = \frac{\bar{X} - \mu_{\bar{X}}}{\frac{\sigma}{\sqrt{n}}}$$

Test statistic for mean

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2}$$

Test statistic for standard deviation

Critical Region

The **critical region** (or **rejection region**) is the set of all values of the test statistic that cause us to reject the null hypothesis.

Significance Level

The **significance level** (denoted by α) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true. Common choices for α are 0.05, 0.01, and 0.10.

Critical Value

A **critical value** is any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to rejection of the null hypothesis. The critical values depend on the nature of the null hypothesis, the sampling distribution that applies, and the significance level α .

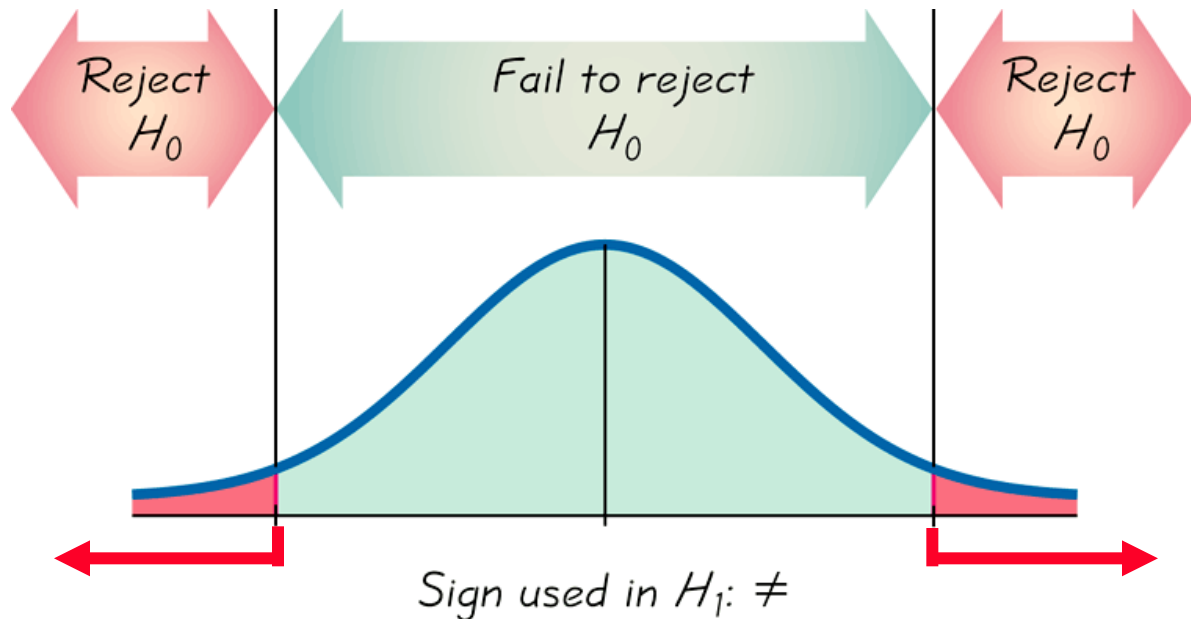
Two-tailed, Right-tailed, Left-tailed Tests

The **tails** in a distribution are the extreme regions bounded by critical values.

Two-tailed Test

$H_0: =$ α is divided equally between
the two tails of the critical
region
 $H_1: \neq$

Means less than or greater than

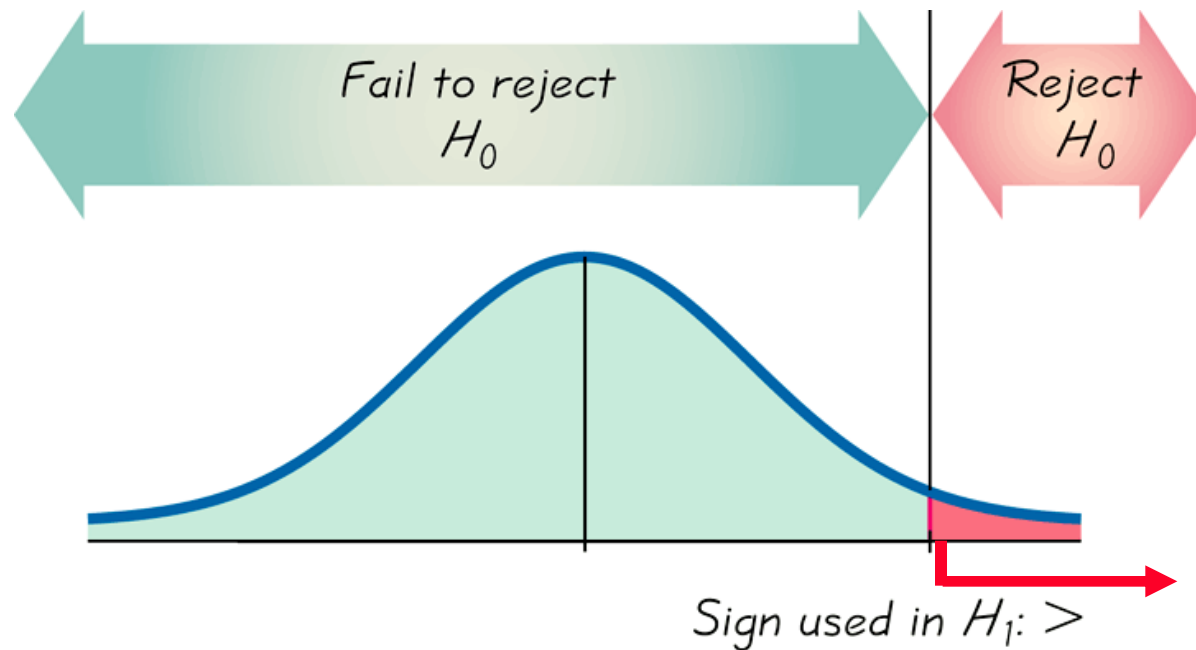


Right-tailed Test

$$H_0: =$$

$$H_1: >$$

Points Right

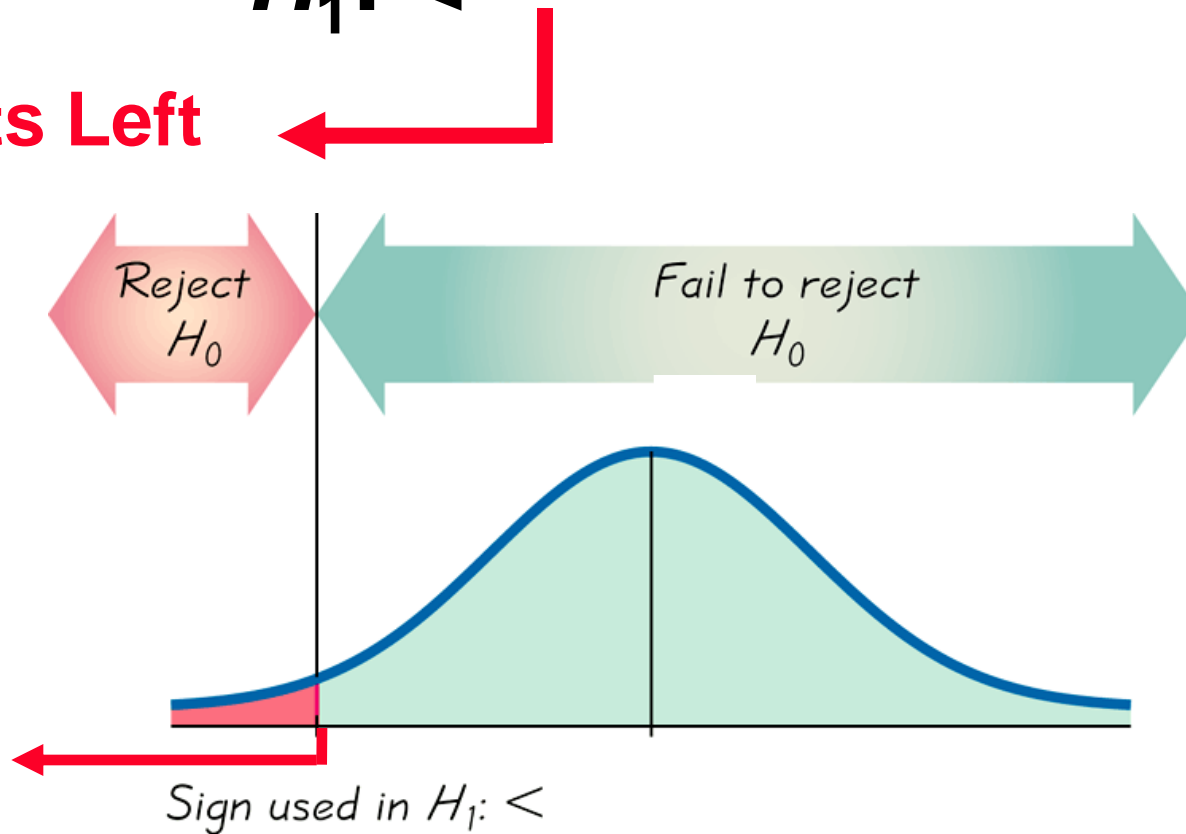


Left-tailed Test

$$H_0: =$$

$$H_1: <$$

Points Left



P-Value

The ***P*-value** (or ***p*-value** or **probability value**) is the probability of getting a value of the test statistic that is **at least as extreme** as the one representing the sample data, assuming that the null hypothesis is true. The null hypothesis is rejected if the ***P*-value** is very small, such as 0.05 or less.

Conclusions in Hypothesis Testing

**We always test the null hypothesis.
The initial conclusion will always be
one of the following:**

- 1. Reject the null hypothesis.**
- 2. Fail to reject the null hypothesis.**

Decision Criterion

Traditional method:

Reject H_0 if the test statistic falls within the critical region.

Fail to reject H_0 if the test statistic does not fall within the critical region.

Decision Criterion - cont

***P*-value method:**

Reject H_0 if the *P*-value $\leq \alpha$ (where α is the significance level, such as 0.05).

Fail to reject H_0 if the *P*-value $> \alpha$.

Decision Criterion - cont

Another option:

Instead of using a significance level such as 0.05, simply identify the *P*-value and leave the decision to the reader.