Statistical Inference: Significance Tests About Hypotheses

Significance Tests About Proportions

An Example

How Do We Interpret the P-value?

- **A significance test analyzes the strength of the evidence *against* the null hypothesis**
- **We start by presuming that H_0 is true**
- **The *burden of proof* is on H_a**

How Do We Interpret the P-value?

- **The approach used in hypothesis testing is called a *proof by contradiction***
- **To convince ourselves that H_a is true, we must show that data contradict H_0**
- ***If the P-value is small, the data contradict H_0 and support H_a***

Two-Sided Significance Tests

- **A two-sided alternative hypothesis has the form H_a: $p \neq p_0$**
- **The P-value is the *two-tail* probability under the standard normal curve**
- **We calculate this by finding the tail probability in a single tail and then doubling it**

Dog Example

- **Study: investigate whether dogs can be trained to distinguish a patient with bladder cancer by smelling compounds released in the patient’s urine**

- **Experiment:**
  - **Each of 6 dogs was tested with 9 trials**
  - **In each trial, one urine sample from a bladder cancer patient was randomly place among 6 control urine samples**

- **Results:**

  In a total of 54 trials with the six dogs, the dogs made the correct selection 22 times (a success rate of 0.407)

- **Does this study provide strong evidence that the dogs' predictions were better or worse than with random guessing?**

Step 1: Check the sample size requirement:

- Is the sample size sufficiently large to use the hypothesis test for a population proportion?
  - Is $np_0 > 15$ and $n(1-p_0) > 15$?
  - $54(1/7) = 7.7$ and $54(6/7) = 46.3$
- The first, np_0 is not large enough
  - We will see that the two-sided test is robust when this assumption is not satisfied

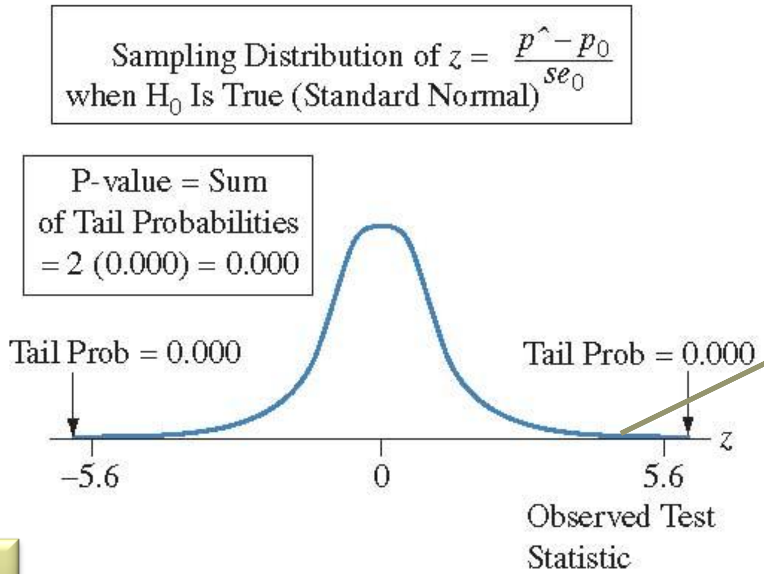
Step 2: Hypotheses

- $H_{0:}$ $p = 1/7$
- $H_{a:}$ $p \neq 1/7$

Step 3: Test Statistic

$$z = \frac{(0.407 - 1/7)}{\sqrt{\frac{(1/7)(6/7)}{54}}} = 5.55$$

Step 4: P-value



Notice that the p-value is zero. This is because the area under the curve for z = 5.6 is .5 by table 4. Any z-score over 3.09 will have an area close to .5.
Hence .5 - .5 = 0.

Calculation of P-value, when $z = 5.6$, for Testing H_0: $p = 1/7$ against H_a: $p \neq 1/7$. Presuming H_0 is true, the P-value is the two-tail probability of a test statistic value even more extreme than observed. **Question:** Is the P-value of 0.000 strong evidence supporting H_0, or strong evidence against H_0?

Step 5: Conclusion

- **Since the P-value is very small and the sample proportion is greater than 1/7, the evidence strongly suggests that the dogs' selections are *better* than random guessing**

- **Insight:**
  - **In this study, the subjects were a *convenience sample* rather than a random sample from some population**
  - **Also, the dogs were not randomly selected**
  - **Any inferential predictions are highly tentative. They are valid only to the extent that the patients and the dogs are representative of their populations**
  - **The predictions become more conclusive if similar results occur in other studies**