

# 2.5

## Simplifying

### OBJECTIVES

- a Multiply a number by 1 to find fraction notation with a specified denominator.
- b Simplify fraction notation.
- c Use the test for equality to determine whether two fractions name the same number.

### Multiplicative Identity for Fractions

When we multiply a number by 1, we get the same number:

$$\frac{3}{5} = \frac{3}{5} \cdot 1 = \frac{3 \cdot 4}{5 \cdot 4} = \frac{12}{20}$$

a Multiply a number by 1 to find fraction notation with a specified denominator.

EXAMPLE A Find a number equivalent to  $\frac{3}{4}$  with a denominator of 36.

**Solution**

Since  $36 \div 4 = 9$ , we multiply by 1, using  $\frac{9}{9}$ :

$$\frac{3}{4} = \frac{3 \cdot 9}{4 \cdot 9} = \frac{3 \cdot 9}{4 \cdot 9} = \frac{27}{36}$$

b Simplify fraction notation.

EXAMPLE C Simplify: 1.  $\frac{28}{35}$  2.  $\frac{3}{24}$  3.  $\frac{8}{32}$

**Solution**

1.  $\frac{28}{35} = \frac{4 \cdot 7}{5 \cdot 7} = \frac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \frac{4}{5}$  Removing a factor equal to 1:  $7/7 = 1$

2.  $\frac{3}{24} = \frac{1 \cdot 3}{8 \cdot 3} = \frac{1 \cdot \cancel{3}}{8 \cdot \cancel{3}} = \frac{1}{8}$  Writing 1 allows for pairing of factors in the numerator and the denominator.

3.  $\frac{8}{32} = \frac{1 \cdot 8}{4 \cdot 8} = \frac{1 \cdot \cancel{8}}{4 \cdot \cancel{8}} = \frac{1}{4}$

a Multiply a number by 1 to find fraction notation with a specified denominator.

EXAMPLE B Find a number equivalent to  $\frac{11}{12}$  with a denominator of 48.

**Solution**

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b Simplify fraction notation.

EXAMPLE D Simplify:  $\frac{210}{245}$

**Solution**

$$\frac{210}{245} = \frac{42 \cdot 5}{49 \cdot 5} = \frac{42 \cdot \cancel{5}}{49 \cdot \cancel{5}} = \frac{42}{49}$$

To find the 42, we divided 210 by 5.

To find the 49, we divided 245 by 5.

The fraction is not simplified since common factors remain in the numerator and denominator.

$$\frac{210}{245} = \frac{42}{49} = \frac{6 \cdot 7}{7 \cdot 7} = \frac{6 \cdot \cancel{7}}{7 \cdot \cancel{7}} = \frac{6}{7}$$

To find the 6, we divided 42 by 7.

To find the 7, we divided 49 by 7.

b Simplify fraction notation.

**Canceling**

Canceling is a shortcut that you may have used for removing a factor that equals 1 when working with fraction notation.

Canceling may be done only when removing common factors in numerators and denominators.

Canceling must be done with care and understanding.

## STUDY TIP

### CAUTION!

The difficulty with canceling is that it is often applied incorrectly in situations like the following:

$$\frac{\cancel{2} + 3}{\cancel{2}} = 3; \quad \frac{\cancel{4} + 1}{\cancel{4} + 2} = \frac{1}{2}; \quad \frac{\cancel{15}}{\cancel{5}4} = \frac{1}{4}$$

The correct answers are:

$$\frac{2 + 3}{2} = \frac{5}{2}; \quad \frac{4 + 1}{4 + 2} = \frac{5}{6}; \quad \frac{15}{54} = \frac{5}{18}$$

In each of the incorrect cancellations, the numbers canceled did not form a factor equal to 1. Factors are parts of products, but in  $2 + 3$ , the numbers 2 and 3 are terms. You cannot cancel terms.

## A Test for Equality

We multiply these two numbers:  $3 \cdot 4$

We multiply these two numbers  $6 \cdot 2$ .

$$\frac{3}{6} \stackrel{\times}{=} \frac{2}{4}$$

We call  $3 \cdot 4$  and  $6 \cdot 2$  **cross products**. Since the cross products are the same  $3 \cdot 4 = 6 \cdot 2$  we know that

$$\frac{3}{6} = \frac{2}{4}$$

**C** Use the test for equality to determine whether two fractions name the same number.

**EXAMPLE F** Use = or  $\neq$  for  $\square$  to write a true sentence:

**Solution**

$$\frac{3}{8} \square \frac{27}{72}$$

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**C** Use the test for equality to determine whether two fractions name the same number.

When denominators are the same, we say that the fractions have a common denominator. One way to compare fractions is to find a common denominator and compare numerators.

**C** Use the test for equality to determine whether two fractions name the same number.

**EXAMPLE E** Use = or  $\neq$  for  $\square$  to write a true sentence:

**Solution**

$$\frac{7}{8} \square \frac{8}{9}$$

We multiply these two numbers:  $7 \cdot 9 = 63$ .

We multiply these two numbers  $8 \cdot 8 = 64$ .

$$\frac{7}{8} \stackrel{\times}{\neq} \frac{8}{9}$$

Because  $63 \neq 64$  (read "63 is not equal to 64"),

$$\frac{7}{8} \neq \frac{8}{9}$$

# 2.6

## Multiplying, Simplifying, and Applications

### OBJECTIVES

- a** Multiply and simplify using fraction notation.
- b** Solve applied problems involving multiplication of fractions.

**a** Multiply and simplify using fraction notation.

To multiply and simplify:

- Write the products in the numerator and the denominator, but do not carry out the products.
- Factor the numerator and the denominator.
- Factor the fraction to remove a factor of 1.
- Carry out the remaining products.

**a** Multiply and simplify using fraction notation.

**EXAMPLE B** Multiply and simplify:  $\frac{12}{28} \cdot \frac{14}{21}$

**Solution**

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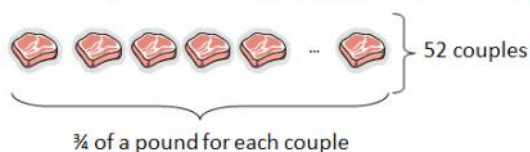
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**Solution**

1. **Familiarize.** Make a drawing or visualize the situation.



3. **Solve.**

$$n = 52 \cdot \frac{3}{4} = \frac{52 \cdot 3}{4}$$

$$= \frac{4 \cdot 13 \cdot 3}{4 \cdot 1} = \frac{4}{4} \cdot \frac{13 \cdot 3}{1} = 39$$

**a** Multiply and simplify using fraction notation.

**EXAMPLE A** Multiply and simplify:  $\frac{4}{5} \cdot \frac{7}{8}$

**Solution**

$$\frac{4}{5} \cdot \frac{7}{8} = \frac{4 \cdot 7}{5 \cdot 8}$$

Note that 4 is a common factor of 4 and 8.

$$= \frac{4 \cdot 7}{5 \cdot 4 \cdot 2}$$

Try to go directly to this step.

$$= \frac{4}{4} \cdot \frac{7}{5 \cdot 2}$$

Removing a factor equal to 1

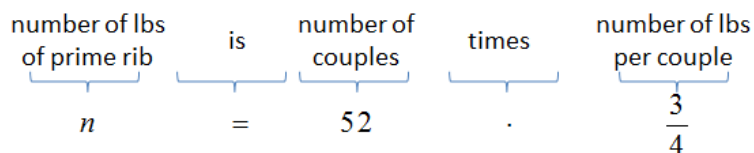
$$= 1 \cdot \frac{7}{10} = \frac{7}{10}$$

**b** Solve applied problems involving multiplication of fractions.

**EXAMPLE C** Catering

Lisa's Gourmet anticipates 52 couples eating prime rib at a banquet. Allowing  $\frac{3}{4}$  lb of prime rib per couple, how many pounds of prime rib should be prepared?

2. **Translate.**



4. **Check.** We could repeat the calculation. We can also think about the reasonableness of the answer. Since each couple requires less than 1 pound, it makes sense that 52 couples require fewer than 52 pounds. This provides a partial check of the answer.

5. **State.**

Lisa's Gourmet should prepare 39 pounds of prime rib.

## 2.7

### Division and Applications

#### OBJECTIVES

- Find the reciprocal of a number.
- Divide and simplify using fraction notation.
- Solve equations of the type  $a \cdot x = b$  and  $x \cdot a = b$  where  $a$  and  $b$  may be fractions.
- Solve applied problems involving division of fractions.

## Reciprocals

If the product of two numbers is 1, we say that they are **reciprocals** of each other. To find the reciprocal of a fraction, interchange the numerator and denominator.

$$\text{Number } \frac{3}{4} \rightarrow \frac{4}{3} \text{ Reciprocal}$$

## 0 Has No Reciprocal

The number 0, or  $\frac{0}{n}$  has no reciprocal.

(Recall that  $\frac{n}{0}$  is not defined.)

**b** Divide and simplify using fraction notation.

**EXAMPLE B** Divide and simplify:  $\frac{3}{4} \div \frac{5}{16}$

$$\begin{aligned} \text{Solution } \frac{3}{4} \div \frac{5}{16} &= \frac{3}{4} \cdot \frac{16}{5} && \text{Multiply by the reciprocal of the divisor} \\ &= \frac{3 \cdot 4 \cdot 4}{4 \cdot 5} && \text{Factoring and identifying a common factor} \\ &= \frac{4 \cdot 3 \cdot 4}{4 \cdot 5} && \text{Removing a factor equal to 1} \\ &= \frac{12}{5} \end{aligned}$$

**c** Solve equations of the type  $a \cdot x = b$  and  $x \cdot a = b$  where  $a$  and  $b$  may be fractions.

**EXAMPLE D** Solve:  $\frac{4}{5}x = \frac{14}{5}$

$$\begin{aligned} \text{Solution } \frac{4}{5}x &= \frac{14}{5} \\ \frac{4}{5}x &= \frac{14}{5} \\ \frac{4}{5}x &= \frac{5}{5} \cdot \frac{14}{5} && \text{Dividing by } 4/5 \text{ on both sides} \\ \frac{4}{5}x &= \frac{4}{5} \cdot \frac{7}{1} \\ x &= \frac{14}{5} \cdot \frac{5}{4} && \text{Multiplying by the reciprocal} \\ x &= \frac{2 \cdot 7 \cdot 5}{2 \cdot 2 \cdot 5} = \frac{2 \cdot 5 \cdot 7}{2 \cdot 5 \cdot 2} = \frac{7}{2} \end{aligned}$$

**d** Solve applied problems involving division of fractions.

**EXAMPLE F** Sofo's Market pre-packages mozzarella cheese in  $\frac{3}{4}$  pound packages. How many packages can be made from a 30-lb block of cheese?

**Solution**

**Familiarize.** Make a drawing. Repeated subtraction or division, will work here.



**a** Find the reciprocal of a number.

**EXAMPLE A** Find the reciprocal: 1.  $\frac{8}{9}$  2.  $\frac{5}{9}$  3.  $\frac{1}{4}$  4.  $\frac{8}{9}$

**Solution**

- The reciprocal of  $\frac{6}{7}$  is  $\frac{7}{6}$ .
- The reciprocal of  $\frac{5}{9}$  is \_\_\_\_\_
- The reciprocal of  $\frac{1}{4}$  is \_\_\_\_\_
- The reciprocal of  $\frac{8}{9}$  is \_\_\_\_\_

## Division of Fractions

To divide fractions, multiply the dividend by the reciprocal of the divisor.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3} = \frac{2 \cdot 4}{5 \cdot 3} = \frac{8}{15}$$

Multiply by the reciprocal of the divisor.

**b** Divide and simplify using fraction notation.

**EXAMPLE C** Divide and simplify:  $\frac{9}{10} \div \frac{13}{15}$

**Solution**

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**c** Solve equations of the type  $a \cdot x = b$  and  $x \cdot a = b$  where  $a$  and  $b$  may be fractions.

**EXAMPLE E** Solve:  $\frac{4}{5}x = 20$

**Solution**  $\frac{4}{5}x = 20$

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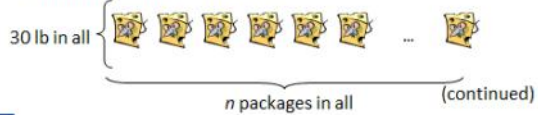
**Translate.** The problem can be translated to the following equation.

$$\frac{3}{4} \cdot n = 30$$

**Solve.**  $\frac{3}{4} \cdot n = 30$

$$\begin{aligned} \frac{4}{3} \cdot \frac{3}{4} \cdot n &= \frac{4}{3} \cdot 30 \\ n &= \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}{3} \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \end{aligned}$$

of division, will work here.



$$n = \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}{3}$$
$$n = \frac{3}{3} \cdot \frac{2 \cdot 2 \cdot 2 \cdot 5}{1}$$
$$n = 40$$

**Check.** If each of the 40 packages took  $\frac{3}{4}$  pound of cheese, we would know that:

$$\frac{3}{4} \cdot 40 = \frac{3 \cdot 4 \cdot 10}{4} = \frac{3 \cdot \cancel{4} \cdot 10}{\cancel{4}} = 30$$

Our answer checks.

**State.** Sofo's Market can fill 40 packages with cheese.

CHAPTER 3 PRETEST

NAME \_\_\_\_\_

CLASS \_\_\_\_\_ SCORE \_\_\_\_\_ GRADE \_\_\_\_\_

	ANSWERS
1. Find the LCM of 24 and 20. [3.1a]	1. _____
2. Use < or > for $\square$ to write a true sentence. [3.3b] $\frac{7}{9} \square \frac{4}{5}$	2. _____
Add or subtract, and simplify.	3. _____
3. $\frac{2}{3} + \frac{5}{6}$ [3.2a]	4. $\frac{5}{6} + \frac{7}{9}$ [3.2a]
5. $\frac{4}{5} - \frac{3}{10}$ [3.3a]	6. $\frac{7}{8} - \frac{5}{6}$ [3.3a]
7. Solve: $x + \frac{3}{8} = \frac{11}{16}$ . [3.3c]	7. _____
8. Convert to fraction notation: $7\frac{5}{8}$ . [3.4a]	8. _____
9. Convert to a mixed numeral: $\frac{11}{2}$ . [3.4a]	9. _____
Perform the indicated calculation. Write a mixed numeral for the answer where appropriate.	10. _____
10. $12\overline{)4789}$ [3.4b]	11. $3\frac{3}{4}$ [3.5a]
	$+ 5\frac{2}{3}$
12. $\begin{array}{r} 12 \\ - 7\frac{5}{8} \\ \hline \end{array}$ [3.5b]	12. _____
13. $3 \cdot 4\frac{8}{15}$ [3.6a]	13. _____

ANSWERS	Perform the indicated calculation. Write a mixed numeral for the answer where appropriate.
14. _____	14. $6\frac{2}{3} \cdot 3\frac{1}{4}$ [3.6a]      15. $98 \div 3\frac{1}{2}$ [3.6b]
15. _____	16. $5\frac{5}{12} \div 3\frac{1}{4}$ [3.6b]
16. _____	Solve.
17. _____	17. At Happy Hollow Camp, the cook bought 100 lb of potatoes and used $78\frac{3}{4}$ lb. How many pounds were left over? [3.5c]
18. _____	18. The weight of water is $62\frac{1}{2}$ lb per cubic foot. How many cubic feet would be occupied by 500 lb of water? [3.6c]
19. _____	19. A courier drove $214\frac{3}{10}$ mi one day and $136\frac{9}{10}$ mi the next. How far did she travel in all? [3.5c]
20. _____	20. A cake recipe calls for $3\frac{3}{4}$ cups of flour. How much flour would be used to make 6 cakes? [3.6c]
21. _____	Estimate each of the following as 0, $\frac{1}{2}$ , or 1. [3.7b]
22. _____	21. $\frac{11}{10}$ 22. $\frac{2}{41}$
23. _____	Estimate each of the following by estimating each mixed numeral as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$ and by estimating each fraction as 0, $\frac{1}{2}$ , or 1. [3.7b]
24. _____	23. $420 \div 13\frac{2}{3}$ 24. $33\frac{14}{15} + 28\frac{3}{4} - 4\frac{25}{28} + \frac{9}{10}$