

3.6 Multiplication and Division Using Mixed Numerals; Applications

OBJECTIVES	
a	Multiply using mixed numerals.
b	Divide using mixed numerals.
c	Solve applied problems involving multiplication and division with mixed numerals.

Multiplication using Mixed Numerals
To multiply using mixed numerals, first convert to fraction notation and multiply. Then convert the answer to a mixed numeral, if appropriate.

a Multiply using mixed numerals.
EXAMPLE B Multiply: $2\frac{1}{7} \cdot 3\frac{3}{5}$
Solution

b Divide using mixed numerals.
EXAMPLE C Divide: $28 \div 1\frac{7}{8}$
Solution
$$28 \div 1\frac{7}{8} = 28 \div \frac{15}{8}$$

$$= 28 \cdot \frac{8}{15}$$

$$= \frac{28 \cdot 8}{1 \cdot 15} = \frac{224}{15} = 14\frac{14}{15}$$

a Multiply using mixed numerals.
EXAMPLE A Multiply: $4\frac{1}{2} \cdot \frac{5}{8}$
Solution
$$4\frac{1}{2} \cdot \frac{5}{8} = \frac{9}{2} \cdot \frac{5}{8} = \frac{45}{16} = 2\frac{13}{16}$$

Note that fraction notation is needed to carry out the multiplication

Division using Mixed Numerals
To divide using mixed numerals, first write fraction notation and divide. Then convert the answer to a mixed numeral, if appropriate.

b Divide using mixed numerals.
EXAMPLE D Divide: $6\frac{1}{4} \div 1\frac{7}{8}$
Solution

C Solve applied problems involving multiplication and division with mixed numerals.

EXAMPLE E Number of Backpacks Made from Material
A backpack requires $2\frac{1}{4}$ yards of material. How many backpacks can be made from $36\frac{3}{4}$ yards of material?

Solution

Familiarize. Draw a picture to help us visualize the situation.



$2\frac{1}{4}$ yards



$36\frac{3}{4}$ yards of material

(continued)

EXAMPLE E Number of Backpacks Made from Material
Check. Check by estimating.

$36 \cdot 2 = 18$ which is close to our answer

State. The number of backpacks that could be made is $16\frac{1}{3}$.

However, only 16 complete backpacks could be made.

EXAMPLE F Area of Shaded Region

Solution

Familiarize. We need to determine what the area of the shaded region is. Recall the area of a rectangle is base times height. To determine the shaded part we subtract the smaller rectangle from the larger rectangle.

$$A_{\text{large}} - A_{\text{small}} = b \cdot h - b \cdot h$$

Solve (continued).

$$A_{\text{large}} - A_{\text{small}} = \frac{725}{4} - \frac{126}{2}$$

$$A_{\text{large}} - A_{\text{small}} = \frac{725}{4} - \frac{126}{2} \cdot \frac{2}{2}$$

$$A_{\text{large}} - A_{\text{small}} = \frac{725 - 252}{4} = \frac{473}{4} = 118\frac{1}{4}$$

EXAMPLE E Number of Backpacks Made from Material

Since each backpack takes $2\frac{1}{4}$ yards, the question can be regarded as how many $2\frac{1}{4}$ are in $36\frac{3}{4}$?

Let b represent the number of backpacks.

Translate. $b = 36\frac{3}{4} \div 2\frac{1}{4}$

EXAMPLE E Number of Backpacks Made from Material

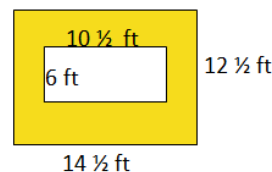
Solve.

$$\begin{aligned} b &= 36\frac{3}{4} \div 2\frac{1}{4} \\ &= \frac{147}{4} \div \frac{9}{4} \\ &= \frac{147}{4} \cdot \frac{4}{9} \\ &= \frac{4}{4} \cdot \frac{147}{9} = \frac{147}{9} = 16\frac{3}{9} = 16\frac{1}{3} \end{aligned}$$

(continued)

C Solve applied problems involving multiplication and division with mixed numerals.

EXAMPLE F What is the area of the shaded region?



EXAMPLE F Area of Shaded Region

Translate.

$$A_{\text{large}} - A_{\text{small}} = 14\frac{1}{2} \cdot 12\frac{1}{2} - 10\frac{1}{2} \cdot 6$$

Solve. We first multiply then subtract:

$$A_{\text{large}} - A_{\text{small}} = 14\frac{1}{2} \cdot 12\frac{1}{2} - 10\frac{1}{2} \cdot 6$$

$$A_{\text{large}} - A_{\text{small}} = \frac{29}{2} \cdot \frac{25}{2} - \frac{21}{2} \cdot \frac{6}{1}$$

(continued)

EXAMPLE F Area of Shaded Region

Check. We could repeat our calculation or estimate our answer. $(14 \cdot 12) \cdot (10 \cdot 6) = 108$

Which is close to our answer.

State.

The area of the shaded region is $118\frac{1}{4}$ sq. ft.

3.7

Order of Operations; Estimation

OBJECTIVES

- a Simplify expressions using the rules for order of operations.
- b Estimate with fraction notation and mixed numerals.

a Simplify expressions using the rules for order of operations.

Rules for Order of Operations

1. Do all calculations within parentheses before operations outside.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

a Simplify expressions using the rules for order of operations.

EXAMPLE A Simplify: $\frac{1}{8} + \frac{3}{4} \div \frac{1}{4} \cdot \frac{3}{8}$

Solution

$$\begin{aligned} \frac{1}{8} + \frac{3}{4} \div \frac{1}{4} \cdot \frac{3}{8} &= \frac{1}{8} + \frac{3}{4} \cdot \frac{4}{1} \cdot \frac{3}{8} && \text{Doing the division first by multiplying by the reciprocal.} \\ &= \frac{1}{8} + \frac{3 \cdot 4 \cdot 3}{4 \cdot 1 \cdot 8} && \text{Doing the multiplications in order from left to right.} \\ &= \frac{1}{8} + \frac{3 \cdot 1 \cdot 3}{1 \cdot 8} && \text{Removing a factor of 1.} \end{aligned}$$

a Simplify expressions using the rules for order of operations.

EXAMPLE A Simplify: $\frac{1}{8} + \frac{3}{4} \div \frac{1}{4} \cdot \frac{3}{8}$

a Simplify expressions using the rules for order of operations.

EXAMPLE B Simplify: $5\frac{7}{12} - \frac{1}{4} \div 3$

Solution

$$\begin{aligned} 5\frac{7}{12} - \frac{1}{4} \div 3 &= 5\frac{7}{12} - \frac{1}{4} \cdot \frac{1}{3} && \text{Divide first (multiply by the reciprocal).} \\ &= 5\frac{7}{12} - \frac{1}{12} && \text{Simplifying.} \end{aligned}$$

EXAMPLE B Simplify: $5\frac{7}{12} - \frac{1}{4} \div 3$

$$\begin{aligned} &= 5\frac{6}{12} && \text{Subtracting.} \\ &= 5\frac{1}{2} && \text{Simplifying.} \end{aligned}$$

a Simplify expressions using the rules for order of operations.

EXAMPLE C Find the Average Foot Length

Tanner measured the lengths of three of his friends feet. They measured $13\frac{1}{2}$ inches, $11\frac{5}{8}$ inches, and $13\frac{3}{4}$ inches. What was the average length of the three friends feet?



EXAMPLE C Find the Average Foot Length

Solution

To compute an average, we add the values and then divide the sum by the number of values. We let f = the average length of the feet.

$$f = \frac{13\frac{1}{2} + 11\frac{5}{8} + 13\frac{3}{4}}{3}$$

EXAMPLE C Find the Average Foot Length

We first add:

$$\begin{aligned} 13\frac{1}{2} + 11\frac{5}{8} + 13\frac{3}{4} &= 13\frac{4}{8} + 11\frac{5}{8} + 13\frac{6}{8} \\ &= 37\frac{11}{8} = 38\frac{3}{8} \end{aligned}$$

EXAMPLE C Find the Average Foot Length

Then we divide:

a Simplify expressions using the rules for order of operations.

EXAMPLE D Simplify: $(5 - 1) \dots (6 - 2)^2$

EXAMPLE C Find the Average Foot Length

Then we divide:

$$f = \frac{38\frac{3}{8}}{3} = 38\frac{3}{8} \div 3 = \frac{307}{8} \cdot \frac{1}{3} = \frac{307}{24} = 12\frac{19}{24}$$

The average length of the friends feet were $12\frac{19}{24}$ in.

a Simplify expressions using the rules for order of operations.

EXAMPLE D Simplify: $\left(\frac{5}{8} - \frac{1}{2}\right) \times 20 + \left(6 + \frac{2}{5}\right)^2$

Solution

$$= \frac{2173}{50} = 43\frac{23}{50}$$

b Estimate with fraction notation and mixed numerals.

EXAMPLE E Estimate each of the following as 0, $\frac{1}{2}$, or 1.

- a. $\frac{3}{19}$ b. $\frac{15}{33}$ c. $\frac{41}{42}$

Solution

a. A fraction is close to 0 when the numerator is small in comparison to the denominator. Thus, 0 is an estimate because 3 is small in comparison to 19.

$$\frac{3}{19} \approx 0$$

b Estimate with fraction notation and mixed numerals.

EXAMPLE E Estimate each of the following as 0, $\frac{1}{2}$, or 1.

- c. $\frac{41}{42}$

c. A fraction is close to 1 when the numerator is nearly equal to the denominator. Thus 1 is an estimate because 41 is nearly equal to 42.

$$\frac{41}{42} \approx 1$$

b Estimate with fraction notation and mixed numerals.

EXAMPLE G Find a number for the box so that $\frac{\square}{75}$ is close to but greater than 0.

Solution

b Estimate with fraction notation and mixed numerals.

EXAMPLE E Estimate each of the following as 0, $\frac{1}{2}$, or 1.

- b. $\frac{15}{33}$

b. A fraction is close to $\frac{1}{2}$ when the denominator is about twice the numerator. Thus $\frac{1}{2}$ is an estimate because $2 \cdot 15 = 30$ and 30 is close to 33.

$$\frac{15}{33} \approx \frac{1}{2}$$

b Estimate with fraction notation and mixed numerals.

EXAMPLE F Find a number for the box so that $\frac{7}{\square}$ is close to but less than 1.

Solution

If the number in the box were 7, we would have 1, so we increase 7 to 8. An answer is 8,

$$\frac{7}{8} \text{ is close to } 1.$$

b Estimate with fraction notation and mixed numerals.

EXAMPLE H Estimate $14\frac{10}{11} + 5\frac{2}{15} - 3\frac{8}{19}$ as a whole number or as a mixed numeral where the fractional part is $\frac{1}{2}$.

Solution

We estimate each fraction as 0, $\frac{1}{2}$, or 1. Then we calculate.

$$\begin{aligned} 14\frac{10}{11} + 5\frac{2}{15} - 3\frac{8}{19} &\approx 15 + 5 - 3\frac{1}{2} \\ &= 20 - 3\frac{1}{2} \\ &= 16\frac{1}{2} \end{aligned}$$

CHAPTER 4 PRETEST

NAME _____

CLASS _____ SCORE _____ GRADE _____

1. Convert to standard notation: 13.6 billion. [4.3b]

2. Write a word name: 2.347. [4.1a]

Write fraction notation. [4.1b]

3. 0.21 4. 9.031

Write decimal notation. [4.1b]

- 379 - .. 43

ANSWERS

1. _____

2. _____

3. _____

4. _____

$$= 16\frac{1}{2}$$

CHAPTER 4 PRETEST

NAME _____

CLASS _____ SCORE _____ GRADE _____

1. Convert to standard notation: 13.6 billion. [4.3b]

2. Write a word name: 2.347. [4.1a]

Write fraction notation. [4.1b]

3. 0.21 4. 9.031

Write decimal notation. [4.1b]

5. $\frac{379}{1000}$ 6. $28\frac{43}{100}$

Which number is larger? [4.1c]

7. 0.035, 0.04 8. 0.099, 0.091

Round 21.0448 to the nearest: [4.1d]

9. Tenth. 10. Thousandth.

11. Add:	12. Subtract:
$\begin{array}{r} 601.3 \\ 5.81 \\ + 0.109 \\ \hline \end{array}$	$\begin{array}{r} 35.612 \\ - 8.476 \\ \hline \end{array}$

Multiply. [4.3a]

13.
$$\begin{array}{r} 0.719 \\ \times 0.25 \\ \hline \end{array}$$
 14. 0.001×324.56

Divide. [4.4a]

15. $3.2 \overline{)130.56}$ 16. $\frac{576.98}{1000}$

Solve.

17. $9.6 \cdot y = 808.896$ [4.4b]18. $x + 0.056 = 12$ [4.2c]

ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____

