9-12-11

Sunday, September 11, 2011
1:15 PM

First do quiz 2 and Chapter 3 pretest.

3.1 Least Common Multiples

OBJECTIVES

a Find the least common multiple, or LCM, of two or more numbers.

Least Common Multiple, LCM

The least common multiple, or LCM, of two natural numbers is the smallest number that is a multiple of both numbers.

a Find the least common multiple, or LCM, of two or more numbers.

Method 1: To find the LCM of a set of numbers using a list of multiples

a) Determine whether the largest number is a multiple of the others. If it is, it is the LCM. That is, if the largest number has the others as factors, the LCM is that number.

b) If not, check multiples of the largest number until you get one that is a multiple of each of the others.

a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE A Find the LCM of 40 and 60.

Solution

a) First list some multiples of 40 by multiplying 40 by 1, 2, 3, and so on: 40, 80, 120, 160, 200, 240, 280, ...

b) Then list some multiples of 60 by multiplying 60 by 1, 2, 3, and so on: 60, 120, 180, 240, ...

c) Now list the numbers common to both lists, the common multiples: 120, 240, ...

d) These are the common multiples of 40 and 60. Which is the smallest? 120

a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE B Find the LCM of 6 and 28.

Solution

1. 28 is not a multiple of 6.

2. Check multiples of 28:

$2 \cdot 28 = 56$

$3 \cdot 28 = 84$ A multiple of 6.

The LCM = 84.

a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE C Find the LCM of 12 and 36.

Solution

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a Find the least common multiple, or LCM, of two or more numbers.

Method 2: To find the LCM of a set of numbers using prime factorizations.

a. Find the prime factorization of each number.

b. Create a product of factors, using each factor the greatest number of times that it occurs in any one factorization.

a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE D Find the LCM of 35 and 90.

Solution

a. Find the prime factorization of each number.

$35 = 5 \cdot 7$ $90 = 2 \cdot 3 \cdot 3 \cdot 5$

b. Create a product by writing factors that appear in the factorizations of 35 and 90, using each the greatest number of times that it occurs in any one factorization.

EXAMPLE D Find the LCM of 35 and 90.

Consider 2: The greatest number of times that 2 occurs in any one factorization is one. We write 2 as a factor one time.

$2 \cdot ?$

Consider 3: The greatest number of times that 3 occurs in any one factorization is two times. We write 3 as the factorizations of 35 and 90, using each the greatest number of times that it occurs in any one factorization.

Consider 3: The greatest number of times that 3 occurs in any one factorization is two times. We write 3 as a factor two times.

$2 \cdot 3 \cdot 3 \cdot ?$

EXAMPLE D Find the LCM of 35 and 90.

$35 = 5 \cdot 7$ $\quad$ $90 = 2 \cdot 3 \cdot 3 \cdot 5$

Consider 5: The greatest number of times that 5 occurs in any one factorization is one time. We write 5 as a factor one time.

times. $2 \cdot 3 \cdot 3 \cdot 5$

Consider 7: The greatest number of times that 7 occurs in any one factorization is one time. We write 7 as a factor one time.

$2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$ The LCM is 630.

EXAMPLE E Find the LCM of 36 and 48.

Solution

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a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE F Find the LCM of 15, 30 and 25.

Solution

a. Write the prime factorization of each number.

$15 = 3 \cdot 5$ $\quad$ $30 = 2 \cdot 3 \cdot 5$ $\quad$ $25 = 5 \cdot 5$

b. Create a product by writing factors, using the greatest number of times that it occurs in any one factorization.

2 occurs as a factor one time

3 occurs as a factor one time

5 occurs as a factor two times

The LCM = $2 \cdot 3 \cdot 5 \cdot 5 = 150$

a Find the least common multiple, or LCM, of two or more numbers.

Method 3: To find the LCM using division by primes:

a. First look for any prime that divides at least two of the numbers with no remainder. Then divide, bringing down any numbers not divisible by the prime.

b. Repeat the process until you can divide no more, that is, until there are no two numbers divisible by the same prime.

a Find the least common multiple, or LCM, of two or more numbers.

EXAMPLE G Find the LCM of 24, 150 and 240.

Solution

2 | 24 150 240
  | 12 75 120

3 | 12 75 120
  | 4 25 40

2 | 4 25 40
  | 2 25 20

2 | 2 25 20
  | 1 25 10

5 | 1 25 10
  | 1 5 2

The LCM

$2 \cdot 3 \cdot 2 \cdot 2 \cdot 5 \cdot 1 \cdot 5 \cdot 2 = 1200$

3.2 Addition and Applications

OBJECTIVES

a Add using fraction notation.

b Solve applied problems involving addition with fraction notation.

a Add using fraction notation.

To add when denominators are the same,

a) add the numerators,

b) keep the denominators,
and

c) simplify, if possible.

$$\frac{2}{6}+\frac{5}{6}=\frac{2+5}{6}=\frac{7}{6}$$

a Add using fraction notation.

To add when denominators are different:

a) Find the least common multiple of the denominators. That number is the least common denominator, LCD.

b) Multiply by 1, using an appropriate notation, n/n, to express each number in terms of the LCD.

c) Add the numerators, keeping the same denominator.

d) Simplify, if possible.

a Add using fraction notation.

EXAMPLE C Add: $\frac{7}{12}+\frac{13}{18}$

Solution

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EXAMPLE E Continued

$$\frac{5}{12}\cdot\frac{3\cdot 7}{3\cdot 7}+\frac{7}{9}\cdot\frac{2\cdot 2\cdot 7}{2\cdot 2\cdot 7}+\frac{8}{21}\cdot\frac{2\cdot 2\cdot 3}{2\cdot 2\cdot 3}$$

In each case, we multiply by 1 to obtain the LCD. To form 1 look at the prime factorization and use the factor(s) missing from each denominator.

b Solve applied problems involving addition with fraction notation.

EXAMPLE F Finding the Sum of Nuts

Logan bought $\frac{1}{2}$ lb of walnuts and $\frac{1}{3}$ lb of almonds. How many pounds of nuts did Logan buy altogether?

Solution

Familiarize. Make a drawing. We let N = the total pounds of nuts.

$\frac{1}{2}$ $\frac{1}{3}$

(continued)

a Add using fraction notation.

EXAMPLE A Add and simplify.

1. $\frac{3}{5}+\frac{1}{5}$ 2. $\frac{5}{16}+\frac{7}{16}$ 3. $\frac{3}{12}+\frac{4}{12}$

1.)____

2.)____

3.)____

a Add using fraction notation.

EXAMPLE B Add: $\frac{1}{5}+\frac{3}{10}$

Solution

a) The LCD is 10. 5 is a factor of 10 so the LCM of 5 and 10 is 10.

b) We need to find a fraction equivalent to $\frac{1}{5}$ with a denominator of 10:

$$\frac{1}{5}+\frac{3}{10}=\frac{1}{5}\cdot\frac{2}{2}+\frac{3}{10}$$

c & d) We add: $\frac{2}{10}+\frac{3}{10}=\frac{5}{10}=\frac{1}{2}.$

a Add using fraction notation.

EXAMPLE E Add: $\frac{5}{12}+\frac{7}{9}+\frac{8}{21}$

Solution

Determine the LCM of 12, 9 and 21:

$12=2\cdot 2\cdot 3$

$9=3\cdot 3$

$21=3\cdot 7$

The LCM is 2·2·3·3·7, or 252

EXAMPLE E Continued

$$\frac{5}{12}\cdot\frac{3\cdot 7}{3\cdot 7}+\frac{7}{9}\cdot\frac{2\cdot 2\cdot 7}{2\cdot 2\cdot 7}+\frac{8}{21}\cdot\frac{2\cdot 2\cdot 3}{2\cdot 2\cdot 3}$$

$$=\frac{5\cdot 21}{12\cdot 21}+\frac{7\cdot 28}{9\cdot 28}+\frac{8\cdot 12}{21\cdot 12}$$

$$=\frac{105}{252}+\frac{196}{252}+\frac{96}{252}$$

$$=\frac{397}{252}$$

EXAMPLE F Finding the Sum of Nuts

Translate.

Walnuts + Almonds = N

$$\frac{1}{2} + \frac{1}{3} = N$$

EXAMPLE F Finding the Sum of Nuts

Solve. $\frac{1}{2}+\frac{1}{3}=N$ $\frac{3}{\ }+\frac{2}{\ }=N$

 

$\frac{1}{2}$ $\frac{1}{3}$

(continued)

EXAMPLE F Finding the Sum of Nuts

Solve.

$$\frac{1}{2}+\frac{1}{3}=N \qquad \frac{3}{6}+\frac{2}{6}=N$$

$$\frac{1}{2}\cdot\frac{3}{3}+\frac{1}{3}\cdot\frac{2}{2}=N \qquad \frac{5}{6}=N$$

Check. Repeat the calculation. The sum should also be larger than either of the individual measurements.

State. Logan bought 5/6 pounds of nuts.