

2.2 Divisibility

OBJECTIVES
a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

Divisibility by 2

A number is **divisible by 2** (is *even*) if it has a ones digit of 0, 2, 4, 6, or 8 (that is, it has an even ones digit).

Divisibility by 3

A number is **divisible by 3** if the sum of its digits is divisible by 3.

Divisibility by 6

A number is **divisible by 6** if its ones digit is 0, 2, 4, 6, or 8 (is even) and the sum of its digits is divisible by 3.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE A Determine whether each of the following numbers is divisible by 2.

- | | |
|---------|---------|
| 1. 457 | 2. 3488 |
| 3. 3200 | 4. 7893 |

Solution

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE B Determine whether the number is divisible by 3. 1. 122.96 2. 303 3. 374

- | | |
|-------|-----------|
| 1. 12 | 1 + 2 = 3 |
|-------|-----------|

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE C Determine whether the number is divisible by 6. 1. 840 2. 90 3. 83

- Solution**
1. 840 is even, divisible by 2. Also $8 + 4 + 0 = 12$, so 840 is divisible by 3, 840 is divisible by 6.

Divisibility 9

A number is **divisible by 9**, if the sum of its digits is divisible by 9.

Divisibility 10

A number is **divisible by 10**, if its ones digit is 0.

Divisibility by 5

A number is **divisible by 5** if its ones digit is 0 or 5.

Divisibility by 4

A number is **divisible by 4** if the number named by its last *two* digits is divisible by 4.

Divisibility by 8

A number is **divisible by 8** if the number named by its last *three* digits is divisible by 8.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE D Determine whether the number is divisible by 9. 1. 4824 2. 524

Solution

1. 4824 $4 + 8 + 2 + 4 = 18$ and 18 is divisible by 9, so 4824 is divisible by 9.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE E Determine whether the number is divisible by 10. 1. 4810 2. 1524

Solution

1. 4810 is divisible by 10 because the ones digit is 0.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE F Determine whether each of the following numbers is divisible by 5.

Solution 1. 340 2. 885 3. 6721

1. 340 is divisible by 5; because its one digit is 0.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE G Determine whether each of the following numbers is divisible by 4.

1. 7732 2. 8453

Solution

1. 7732 is divisible by 4 because 32 is divisible by 4.

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

EXAMPLE H Determine whether each of the following numbers is divisible by 8.

1. 1264 2. 43,911

Solution

1. 1264 is divisible by 8 because 264 is divisible by 8.

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Fractions and Fraction Notation

OBJECTIVES

- a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.
- b Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n .

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

The following are some examples of fractions:

$$\frac{3}{4}, \frac{13}{21}, \frac{-7}{6}, \frac{a}{b}, \frac{3a}{4b}$$

This way of writing number names is called **fraction notation**. The top number is called the **numerator** and the bottom number is called the **denominator**.

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

EXAMPLE B What part of the circle is shaded?

Solution

The object is divided into 8 equal parts.
3 of the parts are shaded



$$\frac{3}{8} \text{ of the circle is shaded.}$$

Fractions as Ratios

A **ratio** is a quotient of two quantities. We can express a ratio with fraction notation.

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

EXAMPLE A Identify the numerator and denominator.

$$\frac{9}{16} \quad \leftarrow \text{Numerator}$$
$$16 \quad \leftarrow \text{Denominator}$$

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

EXAMPLE C What part is shaded?



Solution

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

EXAMPLE D What part of this set, or collection of toys are balls and clowns?

Solution

There are 10 toys in the set.
3 balls are in the set
4 clowns are in the set



$$\text{Balls} = \frac{3}{10}$$

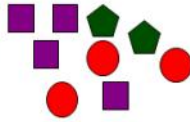
$$\text{Clowns} = \frac{4}{10}$$

a Identify the numerator and the denominator of a fraction; write fraction notation for part of an object.

EXAMPLE E Write Ratios as Fractions

For the set of shapes, what is the ratio of:

1. squares to circles?
2. circles to the total number of shapes?
3. pentagons to squares?
4. total number of shapes to pentagons.



1. _____
2. _____
3. _____
4. _____

The Number 1 in Fraction Notation

$\frac{n}{n} = 1$, for any integer n that is not 0.

b Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n .

EXAMPLE F Simplify: 1. $\frac{7}{7}$ 2. $\frac{-12}{-12}$ 3. $\frac{15x}{15x}$

Solution

The Number 0 in Fraction Notation

$\frac{0}{n} = 0$, for any integer n that is not 0.

b Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n .

EXAMPLE G Simplify: 1. $\frac{0}{12}$ 2. $\frac{0}{-22}$ 3. $\frac{0}{6x}$

Solution

A Denominator of 0

$\frac{n}{0}$ is not defined for any whole number n .

(When asked to simplify $\frac{n}{0}$, we write *undefined*.)

Any Whole Number In Fraction Notation

Any whole number divided by 1 is the whole number.

$\frac{n}{1} = n$, for any whole number n .

b Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n .

EXAMPLE H Simplify: 1. $\frac{8}{1}$ 2. $\frac{-52}{1}$ 3. $\frac{4x}{1}$

Solution

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Multiplication and Applications

OBJECTIVES

- | | |
|----------|---|
| a | Multiply a fraction by a fraction, and multiply a fraction by a whole number. |
| b | Solve applied problems involving multiplication of fractions. |

- a** Multiply a fraction by a fraction, and multiply a fraction by a whole number.

To multiply a fraction by an integer,

- a) multiply the numerator by the whole number, and

$$6 \cdot \frac{4}{5} = \frac{6 \cdot 4}{5} = \frac{24}{5}$$

- b) keep the same denominator.

- a** Multiply a fraction by a fraction, and multiply a fraction by a whole number.

To multiply a fraction by a fraction,

- a) multiply the numerators to get the new numerator, and

$$\frac{9}{7} \cdot \frac{3}{4} = \frac{9 \cdot 3}{7 \cdot 4} = \frac{27}{28}$$

- b) multiply the denominators to get the new denominator.


- b** Solve applied problems involving multiplication of fractions.

EXAMPLE C The length of a plot of land is $\frac{7}{8}$ of a mile and the width is $\frac{2}{3}$ of a mile. What is the area of the land?

Solution

1. **Familiarize.** Recall that area is the length times the width. We make a drawing and let A = the area of the land.

2. **Translate.**

Area is Length times Width $\frac{2}{3}$ 

$$A = \frac{7}{8} \times \frac{2}{3}$$

$\frac{7}{8}$
(continued)

- b** Solve applied problems involving multiplication of fractions.

EXAMPLE D A recipe calls for $\frac{1}{2}$ cup of milk. The baker is making $\frac{1}{2}$ of the recipe. How much milk should the baker use?

Solution

1. **Familiarize.** Make a drawing or visualize the situation. We let n = amount of milk the baker should use.



2. **Translate.**

The sentence $\frac{1}{2} \cdot \frac{1}{2} = n$ corresponds to the situation.

(continued)

- a** Multiply a fraction by a fraction, and multiply a fraction by a whole number.

EXAMPLE A Multiply. 1.) $3 \cdot \frac{4}{5}$ 2.) $\frac{3}{4} \cdot 7$ 3.) $12 \cdot \frac{1}{9}$

Solution

1.) $3 \cdot \frac{4}{5} = \frac{3 \cdot 4}{5} = \frac{12}{5}$

- a** Multiply a fraction by a fraction, and multiply a fraction by a whole number.

EXAMPLE B Multiply 1. $\frac{4}{9} \cdot \frac{4}{5}$ 2. $\frac{3}{4} \cdot \frac{2}{7}$

Solution

1. $\frac{4}{9} \cdot \frac{4}{5} = \frac{4 \cdot 4}{9 \cdot 5} = \frac{16}{45}$

2.

EXAMPLE C Continued

3. **Solve.** $\frac{7}{8} \cdot \frac{2}{3} = \frac{7 \cdot 2}{8 \cdot 3} = \frac{14}{24}$

4. **Check.** We check by repeating the calculation.

5. **State.** The area is $\frac{14}{24} \text{ mi}^2$.

3. **Solve.** $\frac{1}{2} \cdot \frac{1}{2} = \frac{1 \cdot 1}{2 \cdot 2} = \frac{1}{4}$

4. **Check.** We check by repeating the calculation.

5. **State.** The baker should use $\frac{1}{4}$ cup of milk.