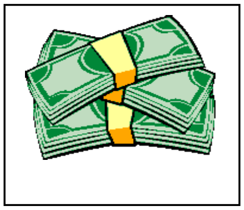
Chapter 4: Finance.

Section 4.1: Simple Interest.



The original amount of money that is lent is called the **principal**, or **present value**. The unpaid principal amount is called the **outstanding principal**, or **balance**. The total amount of money the lender is paid back is called the **future value**. The future value includes both the principal and **interest** paid to the lender for letting you barrow the money. How much interest is paid, depends on the **interest rate**, and the amount of time, the **term**, you need to repay the debt.

Many short-term loans are calculated as a **simple interest**. This means the amount of the interest is computed as a percent-per-year of the principal.

> **Simple Interest Formula**
> The simple interest I on a principal P at an annual rater of interest r for t years is
> $$I = Prt$$

The future value FV is the total of the principal and the interest; therefore,

$FV = P + I = P + Prt = P(1 + r\,t)$.

> **Simple Interest Future Value Formula**
> The future value FV of a principal P at an annual rater of interest r for t years is
> $$FV = P(1 + r\,t)$$

One of the most common uses of simple interest is a short-term, year or less, these loans require a single lump sum payment at the end of the term.

Example 1: For a loan of $2,000.00 at 8% for 3 years, find the simple interest.

Solution: Using the simple interest equation we have: $I = \Pr t = 2000 \bullet .08 \bullet 3 = \$480.00$.

Some institutions still count a year as 360 days and not 365, and a month as 30 days. This is a leftover from the time before calculators and computers. The numbers 360 and 30 were easier to work with. However, when we are converting days into years, we will use a 365-day year.

Example 2: Find the future value of $4,71900 deposited at 4.1% for 11 years.

Solution: FV = P(1 + r t) = 4,719(1 + (4.1/100)11) = $6847.269.

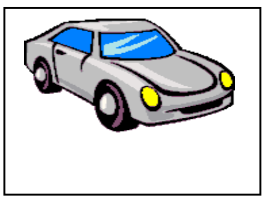
Another similar situation arises when we are calculating the interest over certain months. For example, the 4-month span from January to April does not have the same number of days as the 4-month span from March to June. Also, a written contract signed by the lender and the borrower is called a **loan agreement** or **note**. The **maturity value** of the note refers to the note's future value.

Example 3: How much money must be deposited now at $5\frac{7}{8}$ %interest so that in 2 years and 7 months an account will contain $1,900?

Solution: P = FV/(1+ r t) = 1900/[1 + (.05875)(2 + 7/12)] = 1900/1.15177 = $1649.63.

Note: In this example that the time is 2 years and 7 months, so we must convert the time to years, hence that is why the time is (2 + 7/12).

Purchases made at car dealerships, appliance stores and furniture stores can be financed through the stores. Frequently this type of loan involves **add-on interest**, which consists of a simple interest charge on the loan amount, distributed equally over each payment.

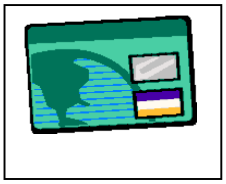


Example 4: # 27,pg. 296 Alice Cohen buys a two-year-old Honda from a car dealer for $9,000. She puts $500 down and finances the

rest through the dealer at 13% add-on interest. If she agrees to make 36 monthly payments, find the size of each payment.

Solution: The total amount that she finances is 9000 – 500 = 8,500. Hence, calculating the future value with a principal of $8,500 we have: FV = 8500[1 + (.13)(36/12)] = $11,815. Hence, her monthly payments are 11815/36 = $328.19.

Purchases with a credit card are subject to a finance charge. However if full payment is received within the grace period, no finance charge is made. The most common way to calculate credit card interest is the **average daily balance**. To find the average daily balance, the balance owed on the account is found for each day in the billing period, and the results are averaged. Simple interest is then charged on the result.



Example 5: # 34 pg. 297. The activity on Charlie Wilson's visa account for one billing period is shown below. Find the average daily balance and the finance charge if the billing period is November 11 through December 10, the previous balance was $642.38, and the annual interest rate is 20%.

November 15	payment	$150.00
November 28	office supplies	$ 23.82
December 01	toy store	$312.58

Solution:

Time Interval	Days	Daily Balance
Nov.11 – Nov.14	4	$642.38
Nov.15 – Nov.27	13	$642.38 - $150.00 = 492.38
Nov.28 – Nov.30	3	492.38 + 23.82 = 516.20
Dec.1 – Dec.10	10	516.20 + 312.58 = 828.78

$$\text{Average Daily Balance} = \frac{4\bullet 642.38 + 13\bullet 492.38 + 3\bullet 516.20 + 10\bullet 828.78}{4+13+3+10} = \$619.93.$$

Next we find the finance charge by $\frac{619.93\bullet .20\bullet 30}{365} = \$9.85$.

Example 6: # 38 pg.298 Sam Needham bought a house from Sheri Silva for $238,300. In lieu of a 20% down payment, Ms. Silva accepted a 10% down payment at the time of the sale and a promissory note from Mr. Needham for an additional 10% due in 4 years. Mr. Needham also agreed to make monthly interest payments to Ms. Silva at 9% interest until the note expires. Mr. Needham obtained a loan from his bank for the remaining 80% of the purchase price. The bank in turn paid Ms. Silva the remaining 80% of the purchase price, less a sales commission (6% of the purchase price) paid to the seller's and buyer's real estate agents.

a.) Find Mr. Needham's down payment.
b.) Find the amount Mr. Needham borrowed from the bank.
c.) Find the amount Mr. Needham borrowed from Ms. Silva.
d.) Find Mr. Needham's monthly interest payment to Ms. Silva.
e.) Find Ms. Silva's total income from all aspects of the down payment (including the down payment, the amount borrowed under the promissory note, and the monthly payments required by the promissory note).
f.) Find Ms. Silva's total income from all aspects of the sale.

Solution: a.) Mr. Needham's down payment is (.10)(238,300) = $23,830.
b.) Mr. Needham borrowed from the bank (.80)(238,300) = $190,640.
c.) The amount Mr. Needham borrowed from Ms. Silva is (.10)(238,300) = $23,830.
d.) Mr. Needham's monthly interest payment to Ms. Silva is $I = P\,r\,t = 23{,}830\,(.09)(4) = \$85{,}788$. So the interest is $85{,}788/(4\bullet 12) = \$178.73$.
e.) Ms. Silva's total income from the down payment is 23,830 + 23,830 + 178.73(48) = $56,239.04
f.) Ms. Silva's income from Mr. Needham's bank is 190,640 – (.06)(238,300) = $176,342.

Section 5.2: Compound Interest.

Many forms of investment earn **compound interest**, in which interest is periodically paid on the existing account balance, which includes both the original principal and interest payments. The **compounding period** (usually annually, semiannually, quarterly, monthly, or daily) refers to the frequency at which interest is computed and deposited. The effects of compound interest is more dramatic than with simple interest.

Compound Interest Formula

At the end of n periods, the future value FV of an initial principal P subject to compound interest at a periodic interest rate i for n periods is

$$FV = P(1+i)^n$$

Example 1: Find the future value of \$36,820 at $7\frac{7}{8}$% compounded quarterly for 4 years.

Solution: FV = $368210\left(1+\dfrac{\frac{63}{800}}{4}\right)^{4\bullet 4} = 36820(1.019)^{16} = \$50,298.76$

Over longer periods of time, compound interest is more profitable than simple interest, because compound interest pays interest on interest. Also, compounding more frequently, daily rather than quarterly, is more profitable to the investor.

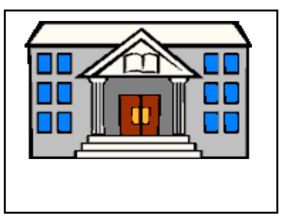
Example 2: What is the interest paid on the amount of principal from example 1?

Solution: Interest paid is = 50298.76 – 36820 = \$13478.76

Example 3: What is the present value needed to generate \$4,459 at $10\frac{3}{4}\%$ compounded quarterly for 4 years?

Solution: $P = \frac{FV}{(1+i)^n} = \frac{4459}{\left(1+\frac{.1075}{4}\right)^{16}} = \$2917.13.$

Which investment is better: one that pays 5.8% compounded daily or one that pays 5.9% compounded quarterly? The best way to compare the investments is by comparing the **annual yield**, or simply **yield**, of a compound interest deposit. How we do this is by comparing the simple interest *rate* that has the same future value the compound *rate* would have in one year. Annual yield provides the consumer with a uniform basis for comparison. The compound rate is sometimes called the **nominal rate** to distinguish it from the yield. Because the future yield of a compound interest is the same as a simple interest future yield we have: $P(1+i)^n = P(1+rt)$.



Example 4: # 28 pg.307 National Trust Savings offers 5-year CDs at 8.25% compounded daily, and Bank of the Future offers 5-year CDs at 8.28% compounded annually. Compute the annual yield for each institution and determine which is more advantageous for the consumer.

Solution: Setting $P(1+i)^n = P(1+rt) \Rightarrow r = (1+i)^n - 1$ we solve for the rate r and n is the number of compounded periods in one year. Hence, National Trust Savings is $\left(1+\frac{.0825}{365}\right)^{365} - 1 = 8.60\%$ where as Bank of the Future is $\left(1+\frac{.0828}{1}\right)^{1} - 1 = 8.28\%$. So National Trust Savings is a better investment.

Remember, the annual yield is the simple interest rate that has the same future value the compound rate would have in one year; it is also the annually compounded rate that has the same future value the nominal rate would have after any amount of time.

Section 5.3: Annuities.