9.5 Angles and Triangles

OBJECTIVES

a Name an angle in five different ways, and given an angle, measure it with a protractor.

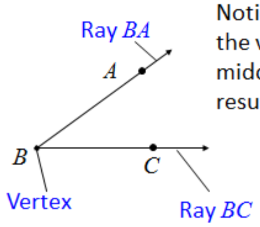
b Classify an angle as right, straight, acute, or obtuse.

c Identify complementary and supplementary angles and find the measure of a complement or a supplement of a given angle.

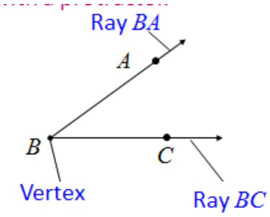
d Classify a triangle as equilateral, isosceles, or scalene, and as right, obtuse, or acute.

e Given two of the angle measures of a triangle, find the third.

An angle is a set of points consisting of two rays, or half-lines, with a common endpoint. The endpoint is called the vertex.



Notice that the name of the vertex is either in the middle or, if no confusion results, listed by itself.

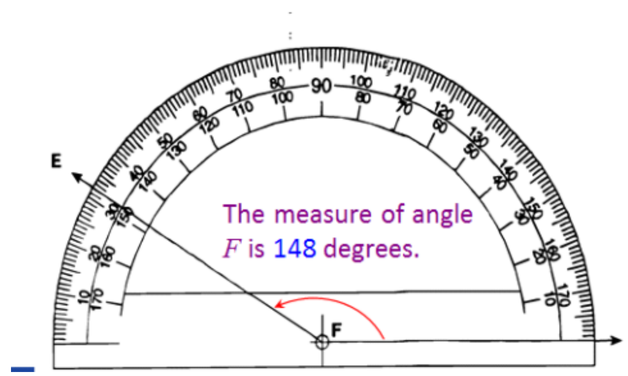


The rays are called the *sides*. The angle above can be named angle ABC, angle CBA, $\angle ABC$, or $\angle CBA$, or $\angle B$.

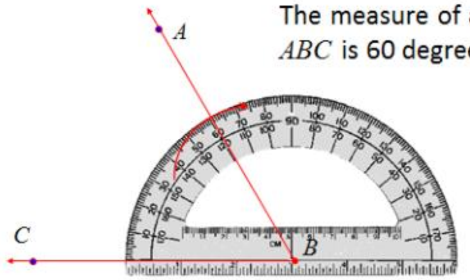
Measuring angles is similar to measuring segments. To indicate the measure of $\angle XYZ$, we write $m\angle XYZ = 65^{\circ}$

A device called a protractor is used to measure angles. Protractors have two scales.

To measure an angle like the angle shown below, we place the protractors (dot) at the vertex and line up one side of the angle's sides at 0 degrees. Then we check where the angle's other side crosses the scale.



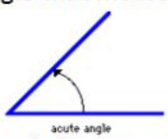
What is the measure of angle ABC?



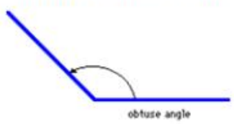
The measure of angle ABC is 60 degrees.

Types of Angles

Acute angle: An angle that measures more than 0° but less than 90°.



Obtuse angle: An angle that measures more than 90° but less than 180°.



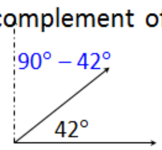
Complementary Angles

Two angles are **complementary** if the sum of their measures is 90°.

Each angle is called a **complement** of the other.

EXAMPLE B Find the measure of the complement of a 42° angle.

Solution



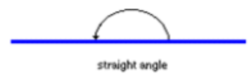
90° – 42° = 48°

The measure of a complement is 48°.

Types of Angles

Right angle: An angle that measures 90°.

Straight angle: An angle that measures 180°.



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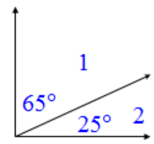
measure of a complement or a supplement of a given angle.

When the sum of the measures of two angles is 90°, the angles are said to be complementary.

$$m\angle 1 + m\angle 2 = 90^\circ$$

$$65^\circ \quad + \quad 25^\circ = 90^\circ$$



measure of a complement or a supplement of a given angle.

EXAMPLE A Identify each pair of complementary angles.

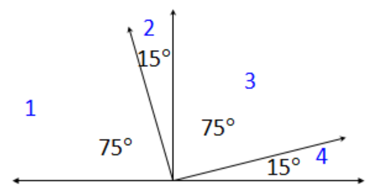
Solution

$\angle 1$ and $\angle 2$

$\angle 1$ and $\angle 4$

$\angle 2$ and $\angle 3$

$\angle 3$ and $\angle 4$



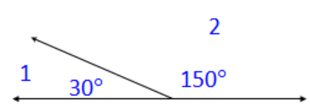
Supplementary Angles

Two angles are **supplementary** if the sum of their measures is 180°.

Each angle is called a **supplement** of the other.

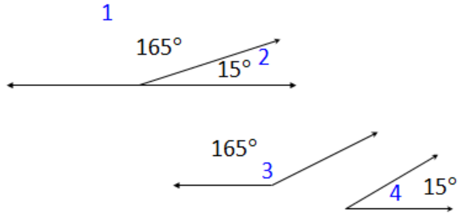
$$m\angle 1 + m\angle 2 = 180^\circ$$

$$30^\circ \quad + \quad 150^\circ = 180^\circ$$



EXAMPLE C Identify each pair of supplementary angles.

Solution



$\angle 1$ and $\angle 2$

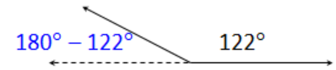
$\angle 1$ and $\angle 4$

$\angle 2$ and $\angle 3$

$\angle 3$ and $\angle 4$

EXAMPLE D Find the measure of a supplement of an angle of 122°.

Solution

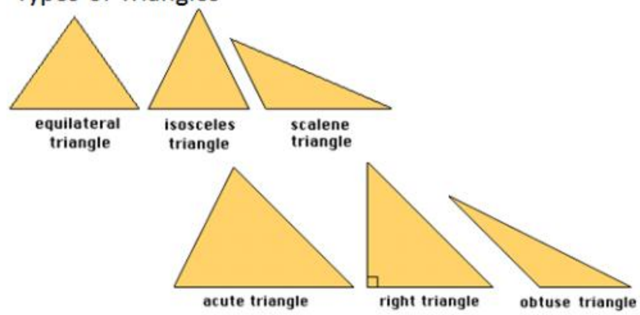


180° – 122° = 58°

The measure of a supplement is 58°.

A triangle is a polygon made up of three segments, or sides. We can classify triangles according to sides and according to angles.

Types of Triangles



Types of Triangles

Equilateral triangle: All sides are the same length.

Isosceles triangle: Two or more sides are the same length.

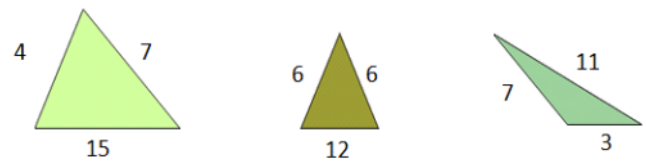
Scalene triangle: All sides are of different lengths.

Right triangle: One angle is a right angle.

Obtuse triangle: One angle is an obtuse angle.

Acute triangle: All three angles are acute.

Identify the type of triangle.



Acute Scalene

Acute Isosceles

Obtuse Scalene

Identify the type of triangle.

Right Scalene

Acute Equilateral

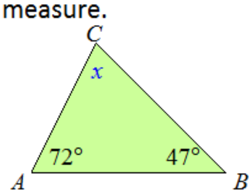
Sum of the Angle Measures of a Triangle

In any triangle ABC, the sum of the measures of the angles is $180°$:

$$m(\angle A)+m(\angle B)+m(\angle C)=180^\circ.$$

EXAMPLE E Find the missing angle measure.

Solution



$$m(\angle A)+m(\angle B)+m(\angle C)=180^\circ$$

$$72^\circ+47^\circ+x=180^\circ$$

$$119^\circ+x=180^\circ$$

$$x=180^\circ-119^\circ$$

$$x=61^\circ$$

The measure of angle C is 61°.

9.6 Square Roots and the Pythagorean Theorem

OBJECTIVES

a Simplify square roots of squares such as $\sqrt{25}$.
b Approximate square roots.
c Given the lengths of any two sides of a right triangle, find the length of the third side.
d Solve applied problems involving right triangles.

Square Root

If a number is a product of two identical factors, then either factor is called a **square root** of the number. (If $a = c^2$, then c is a square root of a.) The symbol $\sqrt{\ }$ (called a **radical sign**) is used in naming square roots.

EXAMPLE A Simplify: $\sqrt{49}$ $\sqrt{169}$ $\sqrt{324}$

Solution

$\sqrt{49}$	$\sqrt{169}$	$\sqrt{324}$
$\sqrt{49} = 7$	$\sqrt{169} = 13$	$\sqrt{324} = 18$
Note that $7^2 = 49$	Note that $13^2 = 169$	Note that $18^2 = 324$

Many square roots can't be written as whole numbers or fractions.

We can use a calculator to find a decimal approximation.

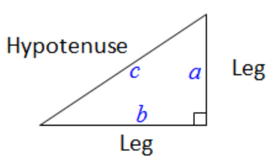
EXAMPLE B Approximate to the nearest thousandth.
$\sqrt{5}$ $\sqrt{32}$ $\sqrt{190}$

Solution

We use a calculator to find each square root. Since more than three decimal places are given, we round back to three places.

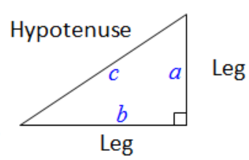
$\sqrt{5}$	$\sqrt{32}$	$\sqrt{190}$
$\sqrt{5} = 2.236$	$\sqrt{32} = 5.657$	$\sqrt{190} = 13.784$

Recall that a right triangle is a triangle with a 90° angle, as shown here.



c Given the lengths of any two sides of a right triangle, find the length of the third side.

In a right triangle, the longest side is called the hypotenuse. It is also opposite the right angle. The other two sides are called legs. We generally use the letters a and b for the lengths of the legs and c for the length of the hypotenuse.

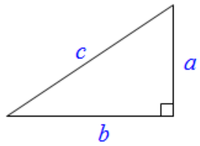


The Pythagorean Theorem

In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then

$a^2 + b^2 = c^2$, or

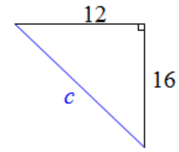
$(\text{Leg})^2 + (\text{Leg})^2 = (\text{Hypotenuse})^2$.



The equation $a^2 + b^2 = c^2$ is called the **Pythagorean equation**.

EXAMPLE C Find the length of the hypotenuse of this right triangle.

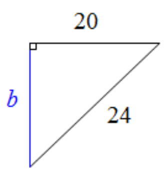
Solution



$$a^2 + b^2 = c^2$$
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$c = \sqrt{400} = 20$$

EXAMPLE D Find Length of b

Find the length b for the right triangle shown. Give an exact answer and an approximation to three decimal places.



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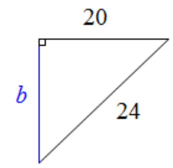
EXAMPLE D Find Length of b.

Solution



$$a^2 + b^2 = c^2$$
$$20^2 + b^2 = 24^2$$
$$400 + b^2 = 576$$
$$b^2 = 176$$

Exact answer: $b = \sqrt{176}$

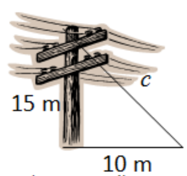
Approximation: $b \approx 13.266$

EXAMPLE E Find Length of Guy Wire

How long must a guy wire be to reach from the top of a 15-m telephone pole to a point on the ground 10 m from the foot of the pole?

Solution

1. Familiarize. We make a drawing and label the known distances. We label the unknown length.



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EXAMPLE E Find Length of Guy Wire

2. Translate. We use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$15^2 + 10^2 = c^2$$

3. Carry out.

$$15^2 + 10^2 = c^2$$
$$225 + 100 = c^2$$
$$325 = c^2$$

Exact answer: $\sqrt{325} = c$

Approximation: $18.028 \approx c$

4. Check.

$$a^2 + b^2 = c^2$$
$$15^2 + 10^2 = 18.028^2$$
$$225 + 100 = 325.001$$

5. State. The guy wire should be about 18.028 m long.