

## 6.4

## Solving Percent Problems Using Proportions

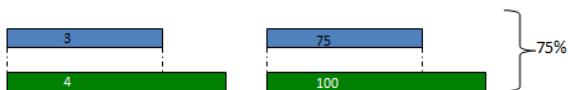
## OBJECTIVES

- a Translate percent problems to proportions.
- b Solve basic percent problems.

## a Translate percent problems to proportions.

A percent is a ratio of some number to 100. For example, 75% is the ratio  $\frac{75}{100}$ .

The numbers 3 and 4 have the same ratio as 75 and 100. Thus,



## a Translate percent problems to proportions.

To solve a percent problem using a proportion, we translate as follows:

$$\begin{array}{l} \text{Number} \rightarrow \frac{N}{100} = \frac{a}{b} \leftarrow \text{Amount} \\ \phantom{\text{Number}} \rightarrow \phantom{\frac{N}{100}} = \phantom{\frac{a}{b}} \leftarrow \text{Base} \end{array}$$

You might find it helpful to read this a "part is to whole as part is to whole."

## a Translate percent problems to proportions.

**EXAMPLE A** Translate to a proportion. 15% of 7 is what?

Solution

15% of 7 is what?

$$\frac{15}{100} = \frac{a}{7}$$

## a Translate percent problems to proportions.

**EXAMPLE B** Translate to a proportion. What is 120% of 73?

Solution

What is 120% of 73?

$$\frac{120}{100} = \frac{a}{73}$$

## a Translate percent problems to proportions.

**EXAMPLE C** Translate to a proportion. 18 is 12% of what?

Solution

18 is 12% of what?

$$\frac{12}{100} = \frac{18}{b}$$

## a Translate percent problems to proportions.

**EXAMPLE D** Translate to a proportion. 35% of what is 21?

Solution

35% of what is 21?

$$\frac{35}{100} = \frac{21}{b}$$

## a Translate percent problems to proportions.

**EXAMPLE E** Translate to a proportion. 22 is what percent of 50?

Solution

22 is what percent of 50?

$$\frac{N}{100} = \frac{22}{50}$$

## a Translate percent problems to proportions.

**EXAMPLE F** Translate to a proportion. What percent of 72 is 9?

Solution

What percent of 72 is 9?

$$\frac{N}{100} = \frac{9}{72}$$

**b** Solve basic percent problems.

**EXAMPLE G** Solve: 35% of what is 21?

Solution 35% of what is 21?

Translate  $\frac{35}{100} = \frac{21}{b}$

Solve  $35 \cdot b = 21 \cdot 100$

$$\frac{35b}{35} = \frac{2100}{35}$$

$b = 60$       35% of **60** is 21

**b** Solve basic percent problems.

**EXAMPLE H** Solve: 15% of 7 is what?

Solution 15% of 7 is what?

Translate  $\frac{15}{100} = \frac{a}{7}$

Solve  $\frac{100a}{100} = \frac{105}{100}$

$a = 1.05$       15% of 7 is **1.05**

**b** Solve basic percent problems.

**EXAMPLE I** Solve: 22 is what percent of 50?

Solution 22 is what percent of 50?

Translate  $\frac{N}{100} = \frac{22}{50}$

Solve  $\frac{50N}{50} = \frac{2200}{50}$

$N = 44$       22 is **44%** of 50

## 6.5

### Applications of Percent

#### OBJECTIVES

- a** Solving applied problems involving percent.
- b** Solve applied problems involving percent of increase or percent of decrease.

**a** Solving applied problems involving percent.

**EXAMPLE A** Percent application.

There have been a total of 54 presidential elections. In some cases a president was re-elected to office accounting for our 43 presidents. The president's who died in office were elected in 13 elections of the 54. In what percent of the elections did the president die?

**EXAMPLE A** Percent application.

Solve.  $\frac{13}{54} = \frac{p \cdot 54}{54}$

$$\frac{13}{54} = p$$

$0.24074 \approx p$

$24.1\% \approx p$

$$\frac{N}{100} = \frac{13}{54}$$

$$N \cdot 54 = 13 \cdot 100$$

$$N \cdot 54 = 1300$$

$$N = \frac{1300}{54}$$

$N = 24.074$

**EXAMPLE A** Percent application.

Familiarize. The question asks for the percent of presidents who died. Estimate 13 is about 1/5 of 54.

Translate. 13 is what percent of 54?

$$13 = p \cdot 54$$

**a** Solving applied problems involving percent.

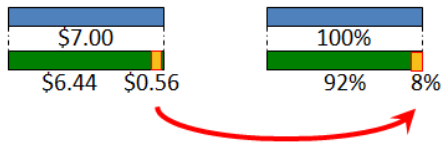
**EXAMPLE A** Percent application.

Check. Our answer is close to our estimate of 1/5 or 20%.

State. About 24% of the U.S. Presidents died in all 54 elections.

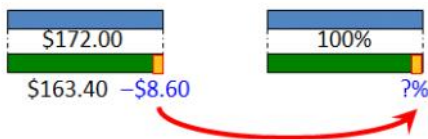
**b** Solve applied problems involving percent of increase or percent of decrease.

What does it mean to say that the price of oil has decreased 8%? If the price was \$7.00 and it went down to \$6.44, then the decrease is \$0.56, which is 8% of the original price.



**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE B** Percent decrease.



$$\begin{array}{r}
 172.00 \text{ Original bill} \\
 -163.40 \text{ New bill} \\
 \hline
 8.60 \text{ Decrease}
 \end{array}$$

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE B** Percent decrease.

Solve. We divide both sides by 172:

$$\begin{aligned}
 \frac{8.60}{172} &= \frac{172p}{172} \\
 0.05 &= p \\
 5\% &= p
 \end{aligned}$$

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE C** Percent increase.

A computer technician earns a starting salary of \$46,000 for one year and receives a 4% raise the following year. What is the new salary?

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE B** Percent decrease.

A family pays a monthly electric bill of \$172.00. With careful monitoring they can reduce their bill to \$163.40. What is the percent of decrease?

Familiarize. We find the amount of decrease and then make a drawing.

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE B** Percent decrease.

Translate. We rephrase and translate.

6.60 is what percent of 172.00?

$$6.60 = p \cdot 172.00$$

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE B** Percent decrease.

Check. To check, we note that, with a 5% decrease, the reduced bill should be 95% of the original bill. Since 95% of 172 = 0.95(172) = 163.40, our answer checks.

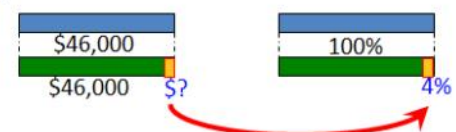
State.

The percent of decrease of the electric bill is 5%.

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE C** Percent increase.

Familiarize. We note that the amount of the raise can be found and then added to the starting salary. A drawing can help us visualize this. We let  $x$  = the new salary.



**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE C** Percent increase.

Translate. We rephrase the question and translate as follows.

What is the starting salary plus 4% of the starting salary?

$$x = 46,000 + 0.04 \cdot 46,000$$

Solve.

$$\begin{aligned}x &= 46,000 + .04(46,000) \\ &= 46,000 + 1840 \\ &= 47,840\end{aligned}$$

[Click to add text](#)

**b** Solve applied problems involving percent of increase or percent of decrease.

**EXAMPLE C** Percent increase.

Check. To check, we note that the new salary is 100% of the starting salary plus 4% of the starting salary. Thus the new salary is 104% of the starting salary. Since  $1.04(46,000) = 47,840$ , our answer checks.

State.

The new salary is \$47,840.

## 6.6

### Sales Tax, Commission, and Discount

#### OBJECTIVES

- a** Solving applied problems involving sales tax and percent.
- b** Solve applied problems involving commission and percent.
- c** Solve applied problems involving discount and percent.

#### Sales Tax

Sales tax computations represent a special type of percent increase problem.

$$\text{Sales Tax} = \text{Sales tax rate} \cdot \text{Purchase price}$$

$$\text{Total price} = \text{Purchase price} + \text{sales tax}$$

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE A** Sales Tax Application

The sales tax rate in Ohio is 6%. How much tax is charged on the purchase of 4 books at \$15.95 each? What is the total price?

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE A** Sales Tax Application

The total price is given by the purchase price plus the sales tax:

$$\$63.80 + \$3.83, \text{ or } \$67.63$$

Check: The total price is 106% of the purchase price. Since  $63.80 \cdot 1.06 = 67.63$ , we have a check.

The sales tax is \$3.83 and the total price is \$67.63

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE A** Sales Tax Application

Solution

a) We first find the cost of the books.

$$4 \cdot \$15.95 = \$63.80$$

b) The sales tax on items costing \$63.80 is

$$\text{Sales tax rate} \cdot \text{Purchase price}$$

$$0.06 \cdot \$63.80 \text{ or } \$3.83$$

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE B** Sales Tax Application

The sales tax is \$140 on the purchase of a new swimming pool which cost \$1750. What is the sales tax rate?

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE B** Sales Tax Application

Solution

Rephrase: Sales tax is what percent of purchase price?

Translate:  $\$140 = p \cdot \$1750$

To solve, divide both sides by 1750

$$\frac{140}{1750} = \frac{p \cdot 1750}{1750} \qquad 0.08 = p$$
$$\frac{140}{1750} = p \qquad 8\% = p$$

The sales tax rate is 8%.

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE C** Sales Tax Application

The sales tax on a digital camera is \$32.18 and the sales tax rate is 7.5%. Find the purchase price (the price before taxes are added).

**a** Solving applied problems involving sales tax and percent.

**EXAMPLE C** Sales Tax Application

Solution

Rephrase: Sales tax is 7.5% of what?

Translate:  $32.18 = 0.075 \cdot x$

To solve, we divide both sides by 0.075:

$$\frac{32.18}{0.075} = \frac{0.075x}{0.075} \qquad 429.07 = x$$
$$\frac{32.18}{0.075} = x \qquad \text{The purchase price is } \$429.07.$$

**Commission**

When you work for a salary, you receive the same amount of money each week or month. When you work for commission, you are paid a percentage of the total sales you complete.

$$\text{Commission} = \text{Commission rate} \cdot \text{Sales}$$

**b** Solve applied problems involving commission and percent.

**EXAMPLE D** Commission Application

A person's sales commission is 8%. What is the commission from the sale of \$73,230 worth of new car sales?

**EXAMPLE D** Commission Application

Solution

Commission = Commission rate · Sales

$$C = 8\% \cdot 73,230$$

$$C = 0.08 \cdot 73,230$$

$$C = 5858.40$$

The commission is \$5858.40.

**b** Solve applied problems involving commission and percent.

**EXAMPLE E** Commission Application

Rebecca earns a commission of \$17,340 from selling a \$289,000 home. What is the commission rate?

**b** Solve applied problems involving commission and percent.

**EXAMPLE E** Commission Application  
Solution

$$\text{Commission} = \text{Commission rate} \cdot \text{Sales}$$
$$17,340 = r \cdot 289,000$$

To solve, we divide both sides by 289,000:

$$\frac{17,340}{289,000} = \frac{289,000r}{289,000}$$
$$0.06 = r$$

6% =  $r$  The commission rate is 6%.

**b** Solve applied problems involving commission and percent.

**EXAMPLE F** Commission Application

Solution

$$\text{Commission} = \text{Commission rate} \cdot \text{Sales}$$
$$1260 = 0.09 \cdot s$$

To solve: divide both sides by 0.09:

$$\frac{1260}{0.09} = \frac{0.09s}{0.09} \quad 14,000 = s$$
$$\frac{126000}{9} = s \quad \text{Sam sold \$14,000 worth of pharmaceuticals.}$$

**b** Solve applied problems involving commission and percent.

**EXAMPLE F** Commission Application

Sam's commission is 9%. She receives a commission of \$1260 on the sale of pharmaceuticals. How much did the pharmaceuticals cost?

**Discount and Sale Price**

Suppose that the regular price of a rug is \$60, and the rug is on sale at 25% off. Since 25% of 60 is \$15, the sale price is \$60 – \$15, or \$45. We call \$60 the original, or marked price, 25% the rate of discount, \$15 the discount, and \$45 the sale price.

$$\text{Discount} = \text{Rate of discount} \cdot \text{Original price}$$

$$\text{Sale Price} = \text{Original price} - \text{Discount}$$

**c** Solve applied problems involving discount and percent.

**EXAMPLE G** Discount Application

A couch marked \$875 is on sale at 25% off. What is the discount? What is the sale price?

**c** Solve applied problems involving discount and percent.

**EXAMPLE G** Discount Application

A couch marked \$875 is on sale at 25% off. What is the discount? What is the sale price?

**c** Solve applied problems involving discount and percent.

**EXAMPLE G** Discount Application

Solution

a) Discount = Rate of discount · Original price

$$D = 25\% \cdot 875$$

$$D = 0.25 \cdot 875$$

$$D = 218.75$$

**c** Solve applied problems involving discount and percent.

**EXAMPLE G** Discount Application

Solution

$$\begin{aligned} \text{b) Sale price} &= \text{Original price} - \text{Discount} \\ &= 875 - 218.75 \\ &= 656.25 \end{aligned}$$

The discount is \$218.75 and the sale price is \$656.25.

**c** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

A dome tent is marked down from \$329 to \$213.85. What is the rate of discount?

**c** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

Solution

Find the discount by subtracting the sale price from the original price:

$$329 - 213.85 = 115.15$$

The discount is 115.15.

**C** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

A dome tent is marked down from \$329 to \$213.85.  
What is the rate of discount?

**C** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

Solution

Find the discount by subtracting the sale price from the original price:

$$329 - 213.85 = 115.15$$

The discount is 115.15.

**C** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

Discount = Rate of discount · Original price

$$115.15 = r \cdot 329$$

To solve, we divide both sides by 329.

$$\frac{115.15}{329} = \frac{r \cdot 329}{329}$$

$$\frac{115.15}{329} = r$$

**C** Solve applied problems involving discount and percent.

**EXAMPLE H** Discount Application

$$0.35 = r$$

$$35\% = r$$

The discount rate is 35%.