

4.1

Decimal Notation, Order, and Rounding

OBJECTIVES

- a Given decimal notation, write a word name.
- b Convert between decimal notation and fraction notation.
- c Given a pair of numbers in decimal notation, tell which is larger.
- d Round decimal notation to the nearest thousandth, hundredth, tenth, one, ten, hundred, or thousand.

- a Given decimal notation, write a word name.

4 2 . 3 2 4 5

The decimal notation 42.3245 means

4 tens + 2 ones + 3 tenths + 2 hundredths + 4 thousandths + 5 ten-thousandths

We read this number as

"Forty-two and three thousand two hundred forty-five ten-thousandths."

The decimal point is read as "and".

To Write a Word Name from Decimal Notation

- a) write a word name for the whole number (the number named to the left of the decimal point),
- 397.685
Three hundred ninety-seven

To Write a Word Name from Decimal Notation

- b) write the word "and" for the decimal point, and
- 397.685
Three hundred ninety-seven and
- c) write a word name for the number named to the right of the decimal point, followed by the place value of the last digit.
- 397.685
Three hundred ninety-seven and six hundred eighty-five thousandths

- a Given decimal notation, write a word name.

EXAMPLE A Write a word name for the number in this sentence: The top women's time for the 50 yard freestyle is 22.62 seconds.

Solution

- a Given decimal notation, write a word name.

EXAMPLE B Write a word name for the number in this sentence:

The one-mile (1.609-km) land speed record is 1,227.985 km/h, set by Andy Green in Thrust SSC in the Black Rock Desert, Nevada, USA, on October 15, 1997.



Solution

One thousand, two hundred twenty-seven and nine hundred eighty-five thousandths

- b Convert between decimal notation and fraction notation.

We can find fraction notation as follows:

$$3.427 = 3 + \frac{427}{1000} = \frac{3000}{1000} + \frac{400}{1000} + \frac{20}{1000} + \frac{7}{1000}$$

Decimal Notation	Fraction Notation
$3.\underline{427}$	$\frac{3427}{1000}$
 <p>3 decimal places</p>	 <p>3 zeros</p>

To Convert from Decimal to Fraction Notation

- a) count the number of decimal places,
- 4.98
↑ 2 places
- b) move the decimal point that many places to the right, and
- 4.98 → Move 2 places.
- c) write the answer over a denominator of 1 followed by that number of zeros
- $\frac{498}{100}$ 2 zeros

b Convert between decimal notation and fraction notation.

EXAMPLE C Write fraction notation for 0.924. Do not simplify.

Solution

b Convert between decimal notation and fraction notation.

EXAMPLE D Write 17.77 as a fraction and as a mixed numeral.

Solution

To write as a mixed numeral, we rewrite the whole number part and express the rest in fraction form:

$$17.77 = 17\frac{77}{100}$$

b Convert between decimal notation and fraction notation.

EXAMPLE D Write 17.77 as a fraction and as a mixed numeral.

Solution

To write as a fraction:

$$17.77 = \frac{1777}{100}$$

$\underbrace{17.77}_{2 \text{ places}}$
 $\underbrace{17.77}_{2 \text{ zeros}} = \frac{1777}{100}$

b Convert between decimal notation and fraction notation.

EXAMPLE E Write decimal notation for $\frac{53}{10}$.

Solution

$$\frac{53}{10} = 5.3$$

$\frac{53}{10}$
↑
1 zero
 $\underbrace{5.3}_{1 \text{ place}}$
 $\frac{53}{10} = 5.3$

b Convert between decimal notation and fraction notation.

EXAMPLE F Write decimal notation for $\frac{134,027}{10,000}$.

Solution

$$\frac{134,027}{10,000} = 13.4027$$

$\frac{134,027}{10,000}$
↑
4 zeros
 $\underbrace{13.4027}_{\text{The decimal point is moved 4 places}}$
 $\frac{134,027}{10,000} = 13.4027$

b Convert between decimal notation and fraction notation.

EXAMPLE G Write decimal notation for $\frac{11}{1000}$.

Solution

$$\frac{11}{1000} = 0.011$$

$\frac{11}{1000}$
↑
3 zeros
 $\underbrace{0.011}_{\text{The decimal point is moved 3 places}}$
 $\frac{11}{1000} = 0.011$

Comparing Two Numbers in Decimal Notation

To compare two numbers in decimal notation, start at the left and compare corresponding digits moving from left to right. If two digits differ, the number with the larger digit is the larger of the two numbers. To ease the comparison, extra zeros can be written to the right of the last decimal place.

c Given a pair of numbers in decimal notation, tell which is larger.

EXAMPLE H Which is larger 3.201 or 3.2?

Solution

3.201	3.201	3.201	3.201
↓	↓	↓	↓
3.2	3.2	3.20	3.200
The same	The same	The same	Different; 1 is larger than 0.

3.201 is larger than 3.2; $3.201 > 3.2$

c Given a pair of numbers in decimal notation, tell which is larger.

EXAMPLE I Which is larger 0.08 or 0.105?

Solution

0.08	0.08
↓	↓
0.105	0.105
The same	Different; 1 is larger than 0.

0.105 is larger $0.105 > 0.08$

Round to a Certain Place

To round to a certain place:

- Locate the digit in that place.
- Consider the next digit to the right.
- If the digit to the right is 5 or greater, round up; if the digit to the right is 4 or lower, round down.

d Round decimal notation to the nearest thousandth, hundredth, tenth, one, ten, hundred, or thousand.

EXAMPLE J Round 0.072 to the nearest tenth.

Solution

- Locate the digit in the tenths place.

$$0.072$$

d Round decimal notation to the nearest thousandth, hundredth, tenth, one, ten, hundred, or thousand.

EXAMPLE K Round 34.7824 to the nearest hundredth.

Solution

- Locate the digit in the hundredths place.

$$34.7824$$

- Consider the next digit to the right.

Solution

- a. Locate the digit in the tenths place.
- b. Consider the next digit to the right. 0.072
- c. Since that digit, 7 is greater than 5, round up 0.1

- b. Consider the next digit to the right. 34.7824
- c. Since that digit, 2 is less than 5, we round down 34.78

d Round decimal notation to the nearest thousandth, hundredth, tenth, one, ten, hundred, or thousand.

EXAMPLE L Round 478.3469 to the nearest thousandth, hundredth, tenth, one, ten, hundred, and thousand.

d Round decimal notation to the nearest thousandth, hundredth, tenth, one, ten, hundred, or thousand.

EXAMPLE L Round 478.3469 to the desired place.

Solution

	4	7	8	.	3	4	6	9
thousandth	4	7	8	.	3	4	7	
hundredth	4	7	8	.	3	5		
tenth	4	7	8	.	3			
one	4	7	8	.				
ten	4	8	0	.				
hundred	5	0	0	.				
thousand			0	.				

4.2

Addition and Subtraction

OBJECTIVES

- a Add using decimal notation.
- b Subtract using decimal notation.
- c Solve equations of the type $x + a = b$ and $a + x = b$ where a and b may be in decimal notation.
- d Balance a checkbook.

a Add using decimal notation.

Adding with decimal notation is similar to adding whole numbers.

First we line up the decimal points so that we can add corresponding place-value digits.

a Add using decimal notation.

Add the digits from the right.

If necessary, we can write extra zeros to the far right of the decimal point so that the number of places is the same.

a Add using decimal notation.

EXAMPLE A Add: $4.31 + 0.146 + 14.2$

Solution Line up the decimal points and write extra zeros.

$$\begin{array}{r}
 4.310 \\
 0.146 \\
 + 14.200 \\
 \hline
 18.656
 \end{array}$$

a Add using decimal notation.

EXAMPLE B Add: $4576 + 17.892$

Solution Line up the decimal points and write extra zeros.

$$\begin{array}{r}
 4576.000 \\
 + 17.892 \\
 \hline
 4593.892
 \end{array}$$

b Subtract using decimal notation.

EXAMPLE C Subtract: $34.07 - 4.0052$

Solution Line up decimal points and subtract, borrowing if necessary.

$$\begin{array}{r} 34.0\overset{6}{7}\overset{9}{0}\overset{10}{0} \\ - 4.0052 \\ \hline 30.0648 \end{array}$$

b Subtract using decimal notation.

EXAMPLE D Subtract $574 - 3.825$.

Solution

$$\begin{array}{r} 574.000 \\ - 3.825 \\ \hline \end{array}$$

c Solve equations of the type $x + a = b$ and $a + x = b$ where a and b may be in decimal notation.

EXAMPLE E Solve $x + 32.78 = 84.19$.

Solution

$$\begin{aligned} x + 32.78 &= 84.19 \\ x + 32.78 - 32.78 &= 84.19 - 32.78 && \text{Subtracting } 32.78 \\ x &= 51.41 && \text{on both sides} \end{aligned}$$

$$\begin{array}{r} 84.19 \\ - 32.78 \\ \hline 51.41 \end{array}$$

c Solve equations of the type $x + a = b$ and $a + x = b$ where a and b may be in decimal notation.

EXAMPLE F Solve $0.0245 + x = 8.0016$

Solution

$$\begin{aligned} 0.0245 + x &= 8.0016 \\ 0.0245 + x - 0.0245 &= 8.0016 - 0.0245 \\ x &= 7.9771 && \text{Subtracting } 0.0245 \\ &&& \text{on both sides} \end{aligned}$$

$$\begin{array}{r} 8.0016 \\ - 0.0245 \\ \hline 7.9771 \end{array}$$

d Balance a checkbook.

EXAMPLE G Find the errors, if any, in the balances in this checkbook.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT							
Date	Check	Transaction	T	Payment	Other	Deposit	Balance
							784.23
9/12	654	Holland Dry Cleaners		19.74			764.49
9/19	655	Circuit Plus		216.48			548.01
9/20		Deposit				326.51	874.55
9/21	656	Food Star		110.24			764.31
9/22	657	Electric Company		126.85			892.16

d Balance a checkbook.

EXAMPLE G Find errors in this checkbook.

Method 1

a) We add the debits:

$$19.74 + 216.48 + 110.24 + 126.85 = 473.31$$

b) We add the deposits/credits: 326.51

d Balance a checkbook.

EXAMPLE G Find the errors in this checkbook.

Method 1 (continued)

c) We add the total of the deposits to the balance brought forward: $784.23 + 326.51 = 1110.74$

d) We subtract the total of the debits:

$$1110.74 - 473.31 = 637.43$$

The result should be the ending balance 892.16.

Since the numbers are not equal, we proceed with method 2.

d Balance a checkbook.

EXAMPLE G Find the errors in this checkbook.

Method 2

We successively add or subtract the deposits/credits and debits, and check the result in the balance column.

d Balance a checkbook.

EXAMPLE G Find the errors in this checkbook.

RECORD ALL CHARGES OR CREDITS THAT AFFECT YOUR ACCOUNT							
Date	Check	Transaction	T	Payment	Other	Deposit	Balance
							784.23
9/12	654	Holland Dry Cleaners		19.74			764.49
9/19	655	Circuit Plus		216.48			548.01
9/20		Deposit				326.51	874.55
9/21	656	Food Star		110.24			764.31
9/22	657	Electric Company		126.85			637.43

Found the mistake, make the correction in the check book.

4.3

Multiplication

OBJECTIVES

- a Multiply using decimal notation.
- b Convert from notation like 45.7 million to standard notation, and convert between dollars and cents.

Multiply Using Decimals

To multiply using decimals: 0.8×0.43

a) Ignore the decimal points, and multiply as though both factors are whole numbers.

$$\begin{array}{r} 0.43 \\ \times 0.8 \\ \hline 344 \end{array}$$

Ignore the decimal points for now.

b) Locate the decimal point in the result. The number of decimal places in the product is the sum of the number of places in the factors. (count places from the right).

$$\begin{array}{r} 0.43 \text{ (2 decimal places)} \\ \times 0.8 \text{ (1 decimal place)} \\ \hline 0.344 \text{ (3 decimal places)} \end{array}$$

a Multiply using decimal notation.

EXAMPLE A Multiply $7.3 \cdot 85.1$.

Solution

Ignore the decimal points and multiply as if both factors are integers then place the decimal point.

$$\begin{array}{r} 85.1 \text{ (1 decimal place)} \\ \cdot 7.3 \text{ (1 decimal place)} \\ \hline 2553 \\ 59570 \\ \hline 621.23 \text{ (2 decimal places)} \end{array}$$

a Multiply using decimal notation.

EXAMPLE B Multiply: $0.0042 \cdot 3215$.

Solution

$$\begin{array}{r} 3215 \text{ (0 decimal places)} \\ \cdot 0.0042 \text{ (4 decimal places)} \\ \hline 6430 \\ 128600 \\ \hline 13.5030 \text{ (4 decimal places)} \end{array}$$

Multiply by 0.1, 0.01, and 0.001

To multiply any number by 0.1, 0.01, 0.001, and so on,

a) count the number of decimal places in the tenth, hundredth, or thousandth, and so on, and

$$0.001 \times 34.45678$$

3 places

$$0.001 \times 34.45678 = 0.03445678$$

Move 3 places to the left.

Multiply by 0.1, 0.01, and 0.001

To multiply any number by 0.1, 0.01, 0.001, and so on,

a) count the number of decimal places in the tenth, hundredth, or thousandth, and so on, and

$$0.001 \times 34.45678$$

3 places

$$0.001 \times 34.45678 = 0.03445678$$

Move 3 places to the left.

Multiply by 0.1, 0.01, 0.001

b) move the decimal point that many places to the left.

$$0.001 \times 34.45678 = 0.03445678$$

a Multiply using decimal notation.

EXAMPLE C Multiply.

1. $0.001 \cdot 6.4$

2. $0.0001 \cdot 721.4$

Solution

1. $0.001 \cdot 6.4 = 0.0064$

Move the decimal point three places to the left. This requires writing extra zeros.

2. $0.0001 \cdot 721.4 = 0.07214$

Move the decimal point four places to the left. This requires writing an extra zero.

Multiply by 10, 100, and 1000

To multiply any number by 10, 100, 1000, and so on,

a) count the number of zeros and

$$1000 \times 34.45678 = 34,456.78$$

3 zeros

b) move the decimal point that many places to the right.

$$1000 \times 34.45678 = 34,456.78$$

Move 3 places to the right.

a Multiply using decimal notation.

EXAMPLE D Multiply.

1. $100 \cdot 4.1$ 2. $1000 \cdot (2.3046)$

Solution

1. $100 \cdot 4.1 = 410$

Move the decimal point two places to the right. The 0 in 410 is a placeholder.

2. $1000 \cdot 2.3046 = 2304.6$

Move the decimal point three places to the right.

Naming Large Numbers

1 hundred = $100 = 10 \cdot 10 = 10^2$
2 zeros

1 thousand = $1000 = 10 \cdot 10 \cdot 10 = 10^3$
3 zeros

1 million = $1,000,000 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6$
6 zeros

Naming Large Numbers

1 billion = $1,000,000,000 = 10^9$
9 zeros

1 trillion = $1,000,000,000,000 = 10^{12}$
12 zeros

EXAMPLE E Convert to Standard Notation

Solution

$6.4 \text{ billion} = 6.4 \cdot 1 \text{ billion}$
 $= 6.4 \cdot 1,000,000,000$ 9 zeros
 $= 6,400,000,000$

Moving the decimal point 9 places to the right

Dollars to Cents Conversions

To convert from dollars to cents, move the decimal point two places to the right and change the \$ sign in front to a ¢ sign at the end.

Cents to Dollars Conversions

To convert from cents to dollars, move the decimal point two places to the left and change the ¢ sign at the end to a \$ in front.

b Convert from notation like 45.7 million to standard notation, and convert between dollars and cents.

EXAMPLE F Convert from dollars to cents.

1. \$142.86 2. \$0.83

Solution

1. $\$142.86 = 14,286¢$

2. $\$0.83 = 83¢$

b Convert from notation like 45.7 million to standard notation, and convert between dollars and cents.

EXAMPLE G Convert from dollars to cents.

1. 8486¢ 2. 343¢

Solution

1. $8486¢ = \$84.86$

2. $343¢ = \$3.43$