

6.1

Percent Notation

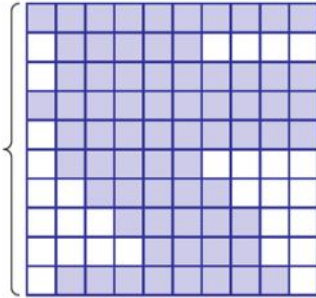
OBJECTIVES

- a Write three kinds of notation for percent.
- b Convert between percent notation and decimal notation.

a Write three kinds of notation for percent.

70 of 100 squares are shaded.

70% or $\frac{70}{100}$ or 0.70 of the large square is shaded.



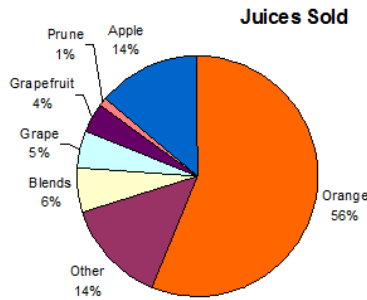
a Write three kinds of notation for percent.

Percent notation is used extensively in our everyday lives. Here are some examples:

- 63% of the aluminum used in the United States is recycled.
- 0.08% blood alcohol level is a standard used by most states at the legal limit for drunk driving.
- 33% of all U.S. citizens say the day they dread most is the day they go to the dentist.

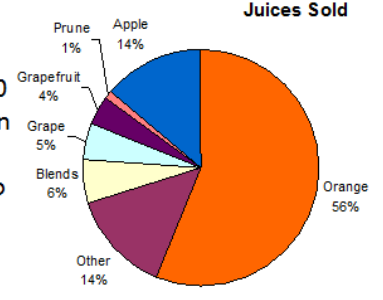
a Write three kinds of notation for percent.

Percent notation is often represented in pie charts to show how the parts of a quantity are related.



a Write three kinds of notation for percent.

To draw the pie chart think of a pie cut into 100 equally sized pieces. Then shade a wedge equal in size to 56 of the pieces to represent 56%. Shade in a wedge equal to 14 of the pieces to represent 14%, and so on.



Percent Notation

The notation $n\%$ means "n per hundred."

Notation for $n\%$

Percent notation, $n\%$, can be expressed using:

ratio $\rightarrow n\% = \text{the ratio of } n \text{ to } 100 = \frac{n}{100},$

fraction notation $\rightarrow n\% = n \times \frac{1}{100},$ or

decimal notation $\rightarrow n\% = n \times 0.01.$

a Write three kinds of notation for percent.

EXAMPLE A Write three kinds of notation for 45%.

Solution

Using ratio: $45\% = \frac{45}{100}$

Using fraction notation: $45\% = 45 \times \frac{1}{100}$

EXAMPLE B Write three kinds of notation for 73.6%.

Solution

Using ratio: $73.6\% = \frac{73.6}{100}$

Using fraction notation: $73.6\% = 73.6 \times \frac{1}{100}$

$$\text{Using fraction notation: } 45\% = 45 \times \frac{1}{100}$$

$$\text{Using decimal notation: } 45\% = 45 \times 0.01$$

$$\text{Using fraction notation: } 73.6\% = 73.6 \times \frac{1}{100}$$

$$\text{Using decimal notation: } 73.6\% = 73.6 \times 0.01$$

b Convert between percent notation and decimal notation.

To write decimal notation for a number like 23%, we can replace the “%” with “· 0.01” and multiply:

$$23\% = 23 \cdot 0.01$$

$$= 0.23$$

Similarly,

$$5.8\% = 5.8 \cdot 0.01$$

$$= 0.058$$

$$324\% = 324 \cdot 0.01$$

$$= 3.24$$

To convert from percent notation to decimal notation:

- a) replace the percent symbol with · 0.01, and
- $$36.5 \times 0.01$$
- b) multiply by 0.01, which means move the decimal point two places to the left.
- $$0.\underline{36}5 \quad \text{Move 2 places to the left}$$
- $$36.5\% = 0.365$$

EXAMPLE C Write an equivalent decimal for 67.34%.
Solution

- a) Replace the % symbol with · 0.01.
- $$67.34 \cdot 0.01$$
- b) Multiply to move the decimal point two places to the left.
- $$0.\underline{67}34$$
- Thus, 67.34% = 0.6734.

EXAMPLE D Write an equivalent decimal for 2.3%.

Solution

- a) Replace the % symbol with · 0.01.
- $$2.3 \cdot 0.01$$
- This zero serves as a placeholder.
- $$0.\underline{02}3$$
- b) Multiply to move the decimal point two places to the left.

Thus, 2.3% = 0.023.

To convert from decimal notation to percent notation:

- Multiply by 100%:
- $$0.675 = 0.675 \times 100\%$$
- That is,
- a) move the decimal point two places to the right, and
- $$0.\underline{67}5 \quad \text{Move 2 places to the right}$$
- b) write a % symbol.
- $$67.5\%$$
- $$0.675 = 67.5\%$$

EXAMPLE E Write percent notation for 0.6.

Solution

- a) Multiply by 100 to move the decimal point two places to the right.
- $$0.\underline{60}$$
- This zero serves as a place holder.

- b) Write a % symbol.

$$60\%$$

Thus 0.6 = 60%

EXAMPLE F Write percent notation for 2.35

Solution

- a) Multiply by 100 to move the decimal point two places to the right.

$$2.\underline{35}$$

- b) Write a % symbol.

$$235\%$$

Thus 2.35 = 235%

6.2

Percent Notation and Fraction Notation

OBJECTIVES

- a Convert from fraction notation to percent notation.
- b Convert from percent notation to fraction notation.

a Convert from fraction notation to percent notation.

To convert from fraction notation to percent notation, $\frac{3}{5}$ Fraction notation

a) find decimal notation by division, and $\begin{array}{r} 0.6 \\ 5 \overline{)3.0} \\ \underline{30} \\ 0 \end{array}$

b) convert the decimal notation to percent notation. $0.6 = 0.60 = 60\%$ percent notation
 $\frac{3}{5} = 60\%$

EXAMPLE B Write $\frac{73}{100}$ as an equivalent percent.

Solution

We use the definition of percent as a ratio.

$$\frac{73}{100} = 73\%$$

EXAMPLE C Write an equivalent fraction for 34% and simplify.

Solution $34\% = \frac{34}{100}$ Using the definition of percent
 $= \frac{2 \cdot 17}{2 \cdot 50}$
 $= \frac{2}{2} \cdot \frac{17}{50}$ Simplify by removing a factor equal to 1.
 $= \frac{17}{50}$

a Convert from fraction notation to percent notation.

EXAMPLE A Write an equivalent percent for $\frac{5}{8}$.
 Solution

a) Find decimal notation by division. $\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$

b) Convert the decimal notation to percent notation. To do so, multiply by 100 to move the decimal point two places to the right, and write a % symbol.

$\frac{5}{8} = 0.625 = 62.5\%$ or $62\frac{1}{2}\%$

To convert from percent notation to fraction notation, 30% Percent notation

a) use the definition of percent as a ratio, and $\frac{30}{100}$
 b) simplify, if possible. $\frac{3}{10}$ Fraction notation

EXAMPLE D Write an equivalent fraction for 122.5% and simplify.

Solution $122.5\% = \frac{122.5}{100}$ Using the definition of percent
 $= \frac{122.5}{100} \times \frac{10}{10}$
 $= \frac{1225}{1000}$

b Convert from percent notation to fraction notation.

EXAMPLE D Write an equivalent fraction for 122.5% and simplify.

Solution

$$\begin{aligned} &= \frac{5}{5} \cdot \frac{245}{200} = \frac{5}{5} \cdot \frac{49}{40} = \frac{49}{40} \\ &= \frac{245}{200} \end{aligned}$$

6.3

Solving Percent Problems Using Percent Equations

OBJECTIVES

- a Translate percent problems to percent equations.
- b Solve basic percent problems.

b Convert from percent notation to fraction notation.

To solve a problem involving percents, it is helpful to translate first to an equation.

$$\begin{array}{cccccc}
 23\% & \text{of} & 5 & \text{is} & \text{what?} & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 23\% & \cdot & 5 & = & a & \text{This is a percent equation.}
 \end{array}$$

$$\begin{array}{cccccc}
 \text{What} & \text{is} & 11\% & \text{of} & 49? & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 a & = & 11\% & \cdot & 49 & \text{Any letter can be used.}
 \end{array}$$

Key Words in Percent

“Of” translates to “.” or “×”.

“Is” translates to “=”.

“What” translates to a variable.

% translates to “ $\times \frac{1}{100}$ ” or “ $\times 0.01$ ”.

EXAMPLE A Translate: What is 19% of 82?

Solution

$$\begin{array}{cccccc}
 \text{What} & \text{is} & 19\% & \text{of} & 82? & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 a & = & 0.19 & \cdot & 82 &
 \end{array}$$

EXAMPLE B Translate: 7 is 10% of what?

Solution

$$\begin{array}{cccccc}
 7 & \text{is} & 10\% & \text{of} & \text{what?} & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 7 & = & 0.10 & \cdot & b &
 \end{array}$$

EXAMPLE C Translate: 18 is what percent of 38?

Solution

$$\begin{array}{cccccc}
 18 & \text{is} & \text{what percent} & \text{of} & 38 & \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
 18 & = & p & \cdot & 38 &
 \end{array}$$

The “Amount” Equation

In solving percent problems, we use the Translate and Solve steps in the problem-solving strategy used throughout the text.

$$\text{Amount} = \text{Percent number} \cdot \text{Base.}$$

b Solve basic percent problems.

3 Types of Percent Problems

1. Finding the amount (the result of taking the percent)

Example: **What** is 25% of 60?

Translation: $a = 0.25 \cdot 60$

2. Finding the base (the number you are taking the percent of)

Example: 15 is 25% of **what number?**

Translation: $15 = 0.25 \cdot b$

(continued)

b Solve basic percent problems.

3 Types of Percent Problems

3. Finding the percent number (the percent itself)

Example: 15 is **what percent** of 60?

Translation: $15 = p \cdot 60$

b Solve basic percent problems.

EXAMPLE D What is 8% of 34?

Solution

Translate: $a = 0.08 \cdot 34$

Solve: The variable is by itself. To solve the equation, we just convert 8% to decimal notation and multiply.

$$a = 0.08(34) \qquad 34$$

$$a = 2.72 \qquad \begin{array}{r} \times 0.08 \\ \hline 2.72 \end{array}$$

Thus, **2.72** is 8% of 34. The answer is 2.72.

b Solve basic percent problems.

EXAMPLE E 15 is 16% of what?

Solution

Translate: 15 is 16% of what?

$$15 = 0.16 \cdot b$$

Solve: To solve we divide both sides

of the equation by 0.16: $15 = 0.16b$

$$\frac{15}{0.16} = \frac{0.16b}{0.16}$$

$$93.75 = b$$

$$\begin{array}{r} 93.75 \\ 0.16 \overline{)15.0000} \\ \underline{144} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

b Solve basic percent problems.

EXAMPLE F 27 is what percent of 36?

Solution

Translate: 27 is what percent of 36?

$$27 = p \cdot 36$$

EXAMPLE F 27 is what percent of 36?

Solve: To solve we divide both sides by 36 and convert the answer to percent notation:

$$27 = p \cdot 36$$

$$\frac{27}{36} = \frac{27p}{36}$$

$$0.75 = p$$

$$p = 75\%$$

$$\begin{array}{r} .75 \\ 36 \overline{)27.00} \\ \underline{252} \\ 180 \\ \underline{180} \\ 0 \end{array}$$