10-19-11

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5.3 Proportions

OBJECTIVES

a Determine whether two pairs of numbers are proportional.

b Solve problems.

a Determine whether two pairs of numbers are proportional.

When two pairs of numbers such as (3, 2 and 6, 4) have the same ratio, we say that they are proportional. The equation

$$\frac{3}{2} = \frac{6}{4}$$

states that the pairs 3, 2 and 6, 4 are proportional. Such an equation is called a proportion.

a Determine whether two pairs of numbers are proportional.

EXAMPLE A Proportional Pairs of Numbers

Determine whether 2, 4, and 5, 10 are proportional.

Solution

We can use cross products to check an equivalent equation:

$$2 \cdot 10 \qquad \frac{2}{4} = \frac{5}{10} \qquad 5 \cdot 4$$

a Determine whether two pairs of numbers are proportional.

EXAMPLE A Proportional Pairs of Numbers

$$2 \cdot 10 \qquad \frac{2}{4} = \frac{5}{10} \qquad 5 \cdot 4$$

$$2 \cdot 10 = 5 \cdot 4$$
$$20 = 20$$

Since the last equation is true, we know that the first equation is also true. The numbers 2, 4 and 5, 10 are proportional.

a Determine whether two pairs of numbers are proportional.

EXAMPLE B Non-Proportional Pairs of Numbers

Determine whether 6, 7 and 8, 9 are proportional.

Solution

$$6 \cdot 9 \qquad \frac{6}{7} = \frac{8}{9} \qquad 8 \cdot 7$$

$$6 \cdot 9 = 8 \cdot 7$$

$$54 \neq 56$$

Since $54 \neq 56$, we know 6, 7 and 8, 9 are not proportional.

a Determine whether two pairs of numbers are proportional.

EXAMPLE C Proportional Pairs of Numbers

Determine whether $2\frac{1}{2}, \frac{1}{2}$, and $4, \frac{4}{5}$ are proportional.

Solution

$$2\tfrac{1}{2} \cdot \tfrac{4}{5} = \tfrac{5}{2} \cdot \tfrac{4}{5} \qquad \frac{2\frac{1}{2}}{\frac{1}{2}} = \frac{4}{\frac{4}{5}} \qquad 4 \cdot \tfrac{1}{2} = 2$$

$$\tfrac{5}{2} \cdot \tfrac{4}{5} = 2$$

$$\tfrac{20}{10} = 2$$

The numbers are proportional.

Solving Proportions

To solve $\frac{x}{a} = \frac{c}{d}$ for any letter, equate *cross products*, and then divide on both sides to get x alone.

b Solve problems.

EXAMPLE D Solving Proportion Problems

Solve: $\frac{x}{8}=\frac{6}{5}$. Write a mixed numeral for the answer.

b Solve problems.

EXAMPLE F Solving Proportion Problems

Solve: $\frac{2.1}{3.6}=\frac{y}{1.8}$. Write decimal notation for the answer.

b Solve problems.

EXAMPLE F Solving Proportion Problems

Solve: $\frac{\frac{18}{5}}{\frac{1}{4}}=\frac{2\frac{1}{2}}{w}$

b Solve problems.

EXAMPLE D Solving Proportion Problems

Solution

$$\frac{x}{8}=\frac{6}{5}$$

$$5\cdot x=6\cdot 8 \quad \text{Equating cross products}$$

$$5x=48$$

$$\frac{5x}{5}=\frac{48}{5} \quad \text{Dividing both sides by 5}$$

$$x=\frac{48}{5} \text{ or } 9\frac{3}{5}$$

Check:

$$\frac{\frac{48}{5}}{8}=\frac{6}{5}$$

$$\frac{48}{5}\cdot 5=48$$

and

$$6\cdot 8=48$$

b Solve problems.

EXAMPLE F Solving Proportion Problems

Solution

$$\frac{2.1}{3.6}=\frac{y}{1.8}$$

$$(2.1)(1.8)=3.6y \quad \text{Equating cross products}$$

$$\frac{(2.1)(1.8)}{3.6}=\frac{3.6y}{3.6} \quad \text{Dividing both sides by 3.6}$$

$$\frac{3.78}{3.6}=y \quad \text{Simplifying}$$

$$1.05=y \quad \text{Dividing}$$

$$\begin{array}{r} 1.05 \\ 3.6\overline{)3.780} \\ \underline{36} \\ 180 \\ \underline{180} \\ 0 \end{array}$$

b Solve problems.

EXAMPLE F Solving Proportion Problems

Solution

$$\frac{\frac{18}{5}}{\frac{1}{4}}=\frac{2\frac{1}{2}}{w}$$

$$\frac{18}{5}\cdot w=\frac{5}{2}\cdot\frac{1}{4} \quad \text{Equating cross products}$$

$$w=\frac{5\cdot 5\cdot 1}{18\cdot 2\cdot 4} \quad \text{Multiply both sides by 5/18.}$$

$$w=\frac{25}{144}$$

5.4 Applications of Proportions

OBJECTIVES

a Solving applied problems involving proportions.

a Solving applied problems involving proportions.

EXAMPLE A Dilution

If 5 ounces of a medicine must be mixed with 8 ounces of water, how many ounces of medicine would be mixed with 36 ounces of water?

a Solving applied problems involving proportions.

EXAMPLE B Ticket Purchases

Maya bought 6 tickets to a cooking show for $73.50. How many tickets can she purchase with $125?

a Solving applied problems involving proportions.

EXAMPLE C Map Scales

On a road atlas, 1 in. represents 22.5 miles. If two cities are 4.5 in. apart on the map, how far apart are they in reality?

a Solving applied problems involving proportions.

EXAMPLE C Map Scales

3. Solve.

$$\frac{1}{22.5}=\frac{4.5}{r}$$

$$1\times r=22.5\times 4.5$$

$$r=101.25 \qquad \frac{1}{22.5}=\frac{4.5}{101.25}$$

$$101.25\times 1=101.25;\; 22.5\times 4.5=101.25$$

a Solving applied problems involving proportions.

EXAMPLE A Dilution

Solution Let m represent how much medicine would be needed.

$$\frac{\text{Medicine needed}}{\text{Ounces of water}} \quad \frac{5}{8}=\frac{m}{36} \quad \frac{\text{How much medicine}}{\text{Ounces of water}}$$

$$5\cdot 36=8\cdot m$$

$$\frac{5\cdot 36}{8}=\frac{8\cdot m}{8}$$

$$\frac{5\cdot \cancel{4}\cdot 9}{\cancel{4}\cdot 2}=m$$

$$22.5=m$$

22.5 ounces of medicine would be needed.

a Solving applied problems involving proportions.

EXAMPLE B Ticket Purchases

Solution Let n = the number of tickets purchased with $125.

$$\frac{\text{Cost}}{\text{Tickets}} \quad \frac{73.50}{6}=\frac{125}{n} \quad \frac{\text{Cost}}{\text{Tickets}}$$

$$73.5n=6\cdot 125$$

$$n=\frac{6\cdot 125}{73.5}$$

$$n\approx 10.2$$

Maya could buy 10 tickets.

a Solving applied problems involving proportions.

EXAMPLE C Map Scales

Solution

1. Familiarize. Let r = the distance apart

2. Translate.

$$\frac{\text{Measure on map}}{\text{Actual distance}} \quad \frac{1}{22.5}=\frac{4.5}{r} \quad \frac{\text{Measure on map}}{\text{Actual distance}}$$

a Solving applied problems involving proportions.

EXAMPLE C Map Scales

4. Check. We substitute into the proportion and check cross products.

$$\frac{1}{22.5}=\frac{4.5}{101.25}$$

$$101.25\times 1=101.25;\; 22.5\times 4.5=101.25$$

The cross products are the same.

5. State. The actual distance apart is 101.25 miles.

a Solving applied problems involving proportions.

EXAMPLE D Estimating Wildlife Populations

To determine the number of deer in a wildlife park, a conservationist catches 275 deer, tags them, and releases them back into the park. Later, 120 deer are caught, and 21 of them are found to be tagged. Estimate how many deer are in the wildlife preserve.

a Solving applied problems involving proportions.

EXAMPLE D Estimating Wildlife Populations

Familiarize. Form two different ratios with can be used to represent the ratio of tagged deer to all the deer.

Translate.

$$\frac{\text{Deer tagged originally}}{\text{Deer in preserve}} \quad \frac{275}{d}=\frac{21}{120} \quad \frac{\text{Tagged deer caught later}}{\text{Deer caught later}}$$

a Solving applied problems involving proportions.

EXAMPLE D Estimating Wildlife Populations

Solve.

$$275\cdot 120 = 21\cdot d$$

$$\frac{275\cdot 120}{21}=\frac{21\cdot d}{21}$$

$$\frac{275\cdot 120}{21}=d$$

$$1571.43 \approx d$$

a Solving applied problems involving proportions.

EXAMPLE D Estimating Wildlife Populations

Check. Substitute into the proportion and check cross products:

$$\frac{275}{1571}=\frac{21}{120} \qquad \begin{aligned}275\cdot 120 &= 33,000\\ 1571\cdot 21 &= 32,991\end{aligned}$$

The cross products are very close.

State.

We estimate that there are 1571 deer in the wildlife preserve.

5.5 Geometric Applications

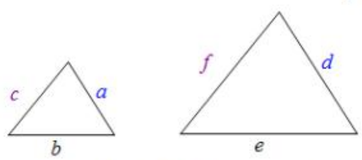
OBJECTIVES

a Find lengths of sides of similar triangles using proportions.

b Use proportions to find lengths in pairs of figures that differ only in size.

a Find lengths of sides of similar triangles using proportions.

The pair of triangles below appear to have the same shape, but their sizes are different. These are examples of *similar triangles*.



Corresponding sides of similar triangles have the same ratio.

$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

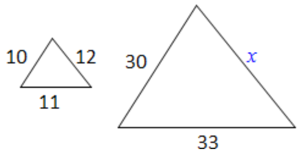
Similar Triangles

Similar triangles have the same shape. The lengths of their corresponding sides have the same ratio—that is, they are proportional.

a Find lengths of sides of similar triangles using proportions.

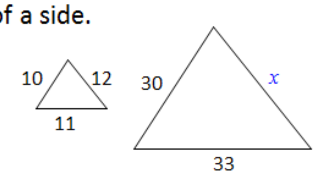
EXAMPLE A Find a measure of a side.

The triangles below are similar triangles. Find the unknown length x.



a Find lengths of sides of similar triangles using proportions.

EXAMPLE A Find a measure of a side.



Solution

The ratio of x to 12 is the same as the ratio of 30 to 10 or 33 to 11.

$$\frac{x}{12}=\frac{30}{10} \text{ and } \frac{x}{12}=\frac{33}{11}$$

a Find lengths of sides of similar triangles using proportions.

EXAMPLE A Find a measure of a side.

Solution

Solve either proportion:

$$\frac{x}{12}=\frac{30}{10} \qquad \frac{x}{12}=3 \qquad x=36$$

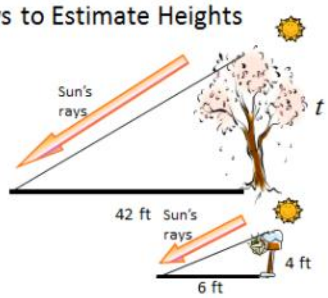
a Find lengths of sides of similar triangles using proportions.

Similar triangles and proportions are often used to compute lengths that would ordinarily be difficult to measure.

a Find lengths of sides of similar triangles using proportions.

EXAMPLE B Using Shadows to Estimate Heights

How tall is a tree that casts a 42-ft shadow at the same time that a 4-ft mailbox casts a 6-ft shadow?



a Find lengths of sides of similar triangles using proportions.

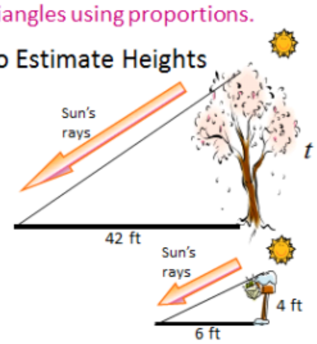
EXAMPLE B Using Shadows to Estimate Heights

Solution

Let t = the height of the tree

$$\frac{\text{Height of tree}}{\text{Length of shadow}}=\frac{\text{Height of mailbox}}{\text{Length of shadow}}$$

$$\frac{t}{42}=\frac{4}{6}$$



a Find lengths of sides of similar triangles using proportions.

EXAMPLE B Using Shadows to Estimate Heights

$$\frac{t}{42}=\frac{4}{6}$$

$t\cdot 6=4\cdot 42$ Equating cross products

$t=\frac{4\cdot 42}{6}$ Dividing both sides by 6

$t=28$ ft

28 feet.

The height of the tree is 28 feet.

b Use proportions to find lengths in pairs of figures that differ only in size.

When one geometric figure is a magnification of another, the figures are similar. Thus, the corresponding lengths are proportional.

b Use proportions to find lengths in pairs of figures that differ only in size.

EXAMPLE C Photo Enlargements

The sides in the enlargements of the pictures below are proportional. Find the width of the larger picture.

b Use proportions to find lengths in pairs of figures that differ only in size.

EXAMPLE C Photo Enlargements

Solution

Let x = the width of the larger picture

$$\frac{\text{Sm photo width}}{\text{Sm photo length}} = \frac{\text{Lg photo width}}{\text{Lg photo length}}$$

$$\frac{12.8}{20.8} = \frac{x}{52}$$

b Use proportions to find lengths in pairs of figures that differ only in size.

EXAMPLE C Photo Enlargements

Solution

$$\frac{12.8}{20.8} = \frac{x}{52}$$

$12.8 \cdot 52 = 20.8 \cdot x$ Equating cross products

$32 = x$ Dividing both sides by 20.8

The width of the larger picture is 32 in.