10-12-11

Sunday, October 09, 2011

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10.3 Subtraction of Real Numbers

OBJECTIVES

a Subtract real numbers and simplify combinations of additions and subtractions.

b Solve applied problems involving addition and subtraction of real numbers.

Subtraction $a - b$

The difference $a - b$ is the number c for which $a = b + c$.

a Subtract real numbers and simplify combinations of additions and subtractions.

EXAMPLE A Subtract 4 − 9.

Solution

Think: 4 − 9 is the number that when added to 9 gives 4.

What number can we add to 9 to get 4?

The number must be negative.

The number is -5:

4 − 9 = −5.

That is, 4 − 9 = −5 because 9 + (−5) = 4.

Subtracting by Adding the Opposite

For any real numbers a and b,

$a - b = a + (-b)$.

(To subtract, add the opposite, or additive inverse, of the number being subtracted.)

a Subtract real numbers and simplify combinations of additions and subtractions.

EXAMPLE B Subtract.

1. −15 − (−25) 2. −13 − 40

Solution

1. −15 − (−25) = −15 + 25 Adding the opposite of −25

= 10

2. −13 − 40 = −13 + (-40) Adding the opposite of 40

= −53

a Subtract real numbers and simplify combinations of additions and subtractions.

EXAMPLE C Subtract. 1. 3 − 7 = 2. −5 − 9

3. −4 − (−10)

Solution

1. 3 − 7 = 3 + (−7)

= −4

The opposite of 7 is −7. We change the subtraction to addition and add the opposite. Instead of subtracting 7, we add −7.

2. −5 − 9 = −5 + (− 9)

= −14

3. −4 − (−10) = −4 + 10

= 6

a Subtract real numbers and simplify combinations of additions and subtractions.

EXAMPLE D Simplify: −4 − (−6) − 10 + 5 − (−7).

Solution

−4 − (−6) − 10 + 5 − (−7)

= −4 + 6 + (−10) + 5 + 7 Adding opposites

= −4 + (−10) + 6 + 5 + 7 Using a commutative law

= −14 + 18

= 4

b Solve applied problems involving addition and subtraction of real numbers.

EXAMPLE E The Johnson's were taking a vacation and one day they drove from mile marker 54 to mile marker 376. How far did they drive?

Solution

376 – 54 = 376 + (–54) Adding the opposite of 54.

= 322 miles

10.4 Multiplication of Real Numbers

OBJECTIVES

a Multiply real numbers.

a Multiply real numbers.

Multiplication of real numbers is like multiplication of arithmetic numbers. The difference is that we must determine whether the answer is positive or negative.

The Product of a Positive and a Negative Number

To multiply a positive number and a negative number, multiply their absolute values. The answer is negative.

a Multiply real numbers.

EXAMPLE A Multiply: 1. (7)(−9) 2. 40(−1) 3. −3 · −7

Solution

1. (7)(−9) = – 63

2. 40(−1) = – 40

3. −3 · −7 = – 21

The Product of Two Negative Numbers

To multiply two negative numbers, multiply their absolute values. The answer is positive.

a Multiply real numbers.

EXAMPLE B Multiply:

1. (−3)(– 4) 2. (– 11)(– 5) 3. (– 2)(– 1)

Solution

1. (– 3)(– 4) = 12

2. (– 11)(– 5)= 55

3. (– 2)(– 1) = 2

To multiply two nonzero real numbers:

a) Multiply the absolute values.

b) If the signs are the same, the answer is positive.

c) If the signs are different, the answer is negative.

a Multiply real numbers.

EXAMPLE C Multiply: 1. −9 · 3(−4)

2. −6 · (−3) · (−4) · (−7)

Solution

1. −9 · 3(−4) = −27(−4) Multiplying the first two numbers

Product of Negative Numbers

The product of an even number of negative numbers is positive.

2. $-6 \cdot (-3) \cdot (-4) \cdot (-7)$

Solution

1. $-9 \cdot 3(-4) = -27(-4)$ Multiplying the first two numbers
$= 108$ Multiplying the results

2. $-6 \cdot (-3) \cdot (-4) \cdot (-7) = 18 \cdot 28$ Each pair of negatives gives a positive product.
$= 504$

The product of an even number of negative numbers is positive.

The product of an odd number of negative numbers is negative.

10.5 Division of Real Numbers and Order of Operations

OBJECTIVES

a Divide integers.

b Find the reciprocal of a real number.

c Divide real numbers.

d Solve applied problems involving multiplication and division of real numbers.

e Simplify expressions using rules for order of operations.

Division

The quotient $a \div b$ or $\frac{a}{b}$, where $b \neq 0$, is that unique real number c for which $a = b \cdot c$.

a Divide integers.

EXAMPLE A Divide, if possible. Check each answer.

1. $15 \div (-3)$
2. $\frac{45}{-5}$

Solution

1. $15 \div (-3) = -5$ *Think:* What number multiplied by -3 gives 15? The number is -5.
Check: $(-3)(-5) = 15$.

2. $\frac{45}{-5} = -9$ *Think:* What number multiplied by -5 gives 45? The number is -9.
Check: $(-5)(-9) = 45$.

Multiply or Divide Two Real Numbers

To multiply or divide two real numbers (where the divisor is nonzero):

a) Multiply or divide the absolute values.

b) If the signs are the same, the answer is positive.

c) If the signs are different, the answer is negative.

Excluding Division by 0

Division by zero is not defined: $a \div 0$, or $\frac{a}{0}$, is not defined for all real numbers a.

Dividends of 0

Zero divided by any nonzero real number is 0:

$$\frac{0}{a} = 0, \qquad a \neq 0.$$

Reciprocals

Two numbers whose product is 1 are called **reciprocals** of each other.

b Find the reciprocal of a real number.

EXAMPLE C Find the reciprocal.

1. $\frac{6}{7}$ 2. -6 3. $\frac{1}{4}$ 4. $-\frac{8}{9}$

Solution

1. The reciprocal of $\frac{6}{7}$ is $\frac{7}{6}$.
2. The reciprocal of −6 is $-\frac{1}{6}$.
3. The reciprocal of $\frac{1}{4}$ is 4.
4. The reciprocal of $-\frac{8}{9}$ is $-\frac{9}{8}$.

a Divide integers.

EXAMPLE B Divide, if possible: −72 ÷ 0.

Solution

$\frac{-72}{0}$ is undefined.

Think: What number multiplied by 0 gives −72? There is no such number because anything times 0 is 0.

Properties of Reciprocals

For $a \neq 0$, the reciprocal of a can be named $\frac{1}{a}$ and the reciprocal of $\frac{1}{a}$ is a.

The reciprocal of any nonzero real number $\frac{a}{b}$ can be named $\frac{b}{a}$.

The number 0 has no reciprocal.

The Sign of a Reciprocal

The reciprocal of a number has the same sign as the number itself.

b Find the reciprocal of a real number.

Number	Opposite (Change the sign.)	Reciprocal (Invert but do not change the sign.)
$-\frac{3}{4}$	$\frac{3}{4}$	$-\frac{4}{3}$
25	−25	$\frac{1}{25}$
$\frac{23}{3}$	$-\frac{23}{3}$	$\frac{3}{23}$
−8.5	8.5	$-\frac{1}{8.5}$ or $-\frac{10}{85}$
0	0	Undefined

Reciprocals and Division

For any real numbers a and b, $b \neq 0$,

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$$

(To divide, multiply by the reciprocal of the divisor.)

c Divide real numbers.

EXAMPLE D Rewrite the division as multiplication.

1. $-9 \cdot 5$ 2. $\frac{3}{4} \div \left(-\frac{2}{5}\right)$

Solution

1. $-9 \cdot 5$ is the same as $-9 \cdot \left(\frac{1}{5}\right)$

2. $\frac{3}{4} \div \left(-\frac{2}{5}\right) = \frac{3}{4}\left(-\frac{5}{2}\right)$

c Divide real numbers.

EXAMPLE E Multiply by Reciprocal

Solution $\frac{3}{4} \div \frac{5}{16} = \frac{3}{4} \cdot \frac{16}{5}$ Multiply by the reciprocal of the divisor

$= \frac{3 \cdot 4 \cdot 4}{4 \cdot 5}$ Factoring and identifying a common factor

$= \frac{4}{4} \cdot \frac{3 \cdot 4}{5}$ Removing a factor equal to 1

$= \frac{12}{5}$

c Divide real numbers.

EXAMPLE E Divide by multiplying by the reciprocal of the divisor.

$\frac{3}{4} \div \frac{5}{16}$

c Divide real numbers.

EXAMPLE F Divide: $-18.6 \div (3)$

Solution

$-18.6 \div (3) =$

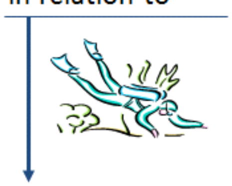
$3\overline{)18.6}$ = 6.2 Do the long division. The answer is negative.

$\frac{-18.6}{3} = -6.2$

d Solve applied problems involving multiplication and division of real numbers.

EXAMPLE G Location of Diver

After diving 110 m below sea level, a diver rises at a rate of 8 meters per minutes for 6 minutes. Where is the diver in relation to the surface?



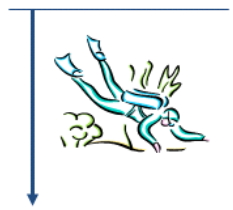
110 m

d Solve applied problems involving multiplication and division of real numbers.

EXAMPLE G Location of Diver

Solution

We first determine by how many meters the diver rose altogether.



110 m

8 meters · 6 = 48 meters

The diver was 110 below sea level and rose 48 meters.

−110 + 48 = −62 meters

Rules for Order of Operations

1. Do all calculations within parentheses, brackets, braces, absolute value symbols, numerators, or denominators.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

e Simplify expressions using rules for order of operations.

EXAMPLE H Simplify.

1. $20 - 12 \div 4 \cdot 2$ 2. $\left|(-3)^3 \div 9\right| - 6(-3)$

Solution

1. $20 - 12 \div 4 \cdot 2$

$20 - 12 \div 4 \cdot 2 = 20 - 3 \cdot 2$

$= 20 - 6$

$= 14$

e Simplify expressions using rules for order of operations.

EXAMPLE H Simplify: 2. $\left|(-3)^3 \div 9\right| - 6(-3)$

2. $\left|(-3)^3 \div 9\right| - 6(-3)$

$$\begin{aligned}\left|(-3)^3 \div 9\right| - 6(-3) &= \left|-27 \div 9\right| - 6(-3) \\ &= \left|-3\right| - 6(-3) \\ &= 3 - 6(-3) \\ &= 3 - (-18) \\ &= 21\end{aligned}$$

e Simplify expressions using rules for order of operations.

EXAMPLE I Simplify: $\dfrac{6 \cdot 3 \div 9}{-2}$.

Solution

$$\begin{aligned}\frac{6 \cdot 3 \div 9}{-2} &= \frac{18 \div 9}{-2} \\ &= \frac{2}{-2} \\ &= -1\end{aligned}$$