10-10-11

Sunday, October 09, 2011
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10.1 The Real Numbers

OBJECTIVES

a	State the integer that corresponds to a real-world situation.
b	Graph rational numbers on the number line.
c	Convert from fraction notation for a rational number to decimal notation.
d	Determine which of two real numbers is greater and indicate which, using < or >.
e	Find the absolute value of a real number.

a State the integer that corresponds to a real-world situation.

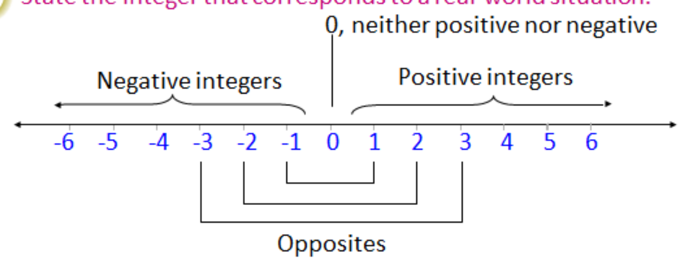
Integers

Integers consist of the whole numbers and their opposites.

Integers

The integers: ..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

a State the integer that corresponds to a real-world situation.



Integers to the left of zero on the number line are called **negative integers** and those to the right of zero are called **positive integers**. Zero is neither positive nor negative and serves as its own opposite.

a State the integer that corresponds to a real-world situation.

EXAMPLE A Tell which integer corresponds to each situation.

1. Death Valley is 282 feet below sea level.
2. Margaret owes $312 on her credit card. She has $520 in her checking account.

Solution

1. 282 below sea level corresponds to − 282.

2. The integers − 312 and 520 correspond to the situation.

b Graph rational numbers on the number line.

Fractions such as ½ are not integers. A larger system called rational numbers contains integers and fractions. The rational numbers consist of quotients of integers with nonzero divisors.

The following are rational numbers:

$$\frac{2}{3},\ -\frac{2}{3},\ \frac{7}{1},\ 4,\ -3,\ 0,\ \frac{23}{-8},\ 2.4,\ -0.17$$

Rational Numbers

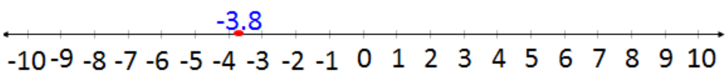
The rational numbers consist of all numbers that can be named in the form $\frac{a}{b}$, where a and b are integers and b is not 0.

(b) Graph rational numbers on the number line.

EXAMPLE C Graph: − 3.8

Solution

The graph of − 3.8 is 8/10 of the way from − 3 to − 4.



(c) Convert from fraction notation for a rational number to decimal notation.

EXAMPLE D Find decimal notation for $-\frac{7}{40}$.

Solution

Because $\frac{7}{40}$ means 7 ÷ 40, we divide.

$$40\overline{)7.000} = 0.175$$

40
300
280
200
200
0

We are finished when the remainder is 0.

$$-\frac{7}{40} = -0.175$$

The Real-Number System

The rational numbers and the irrational numbers together correspond to all the points on a number line and make up what is called the real-number system.

(d) Determine which of two real numbers is greater and indicate which, using < or >.

Numbers are written in order on the number line, increasing as we move to the right. For any two numbers on the line, the one to the left is less than the one to the right.

The symbol < means "is less than,"

−4 < 8 is read "−4 is less than 8."

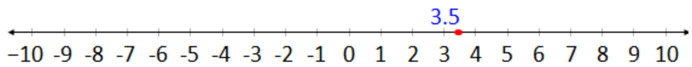
(b) Graph rational numbers on the number line.

EXAMPLE B Graph: $\frac{7}{2}$

To graph a number means to find and mark its point on the number line.

Solution

The number $\frac{7}{2}$ can be named $3\frac{1}{2}$, or 3.5. Its graph is halfway between 3 and 4.



(c) Convert from fraction notation for a rational number to decimal notation.

Each rational number can be named using fraction notation or decimal notation.

(c) Convert from fraction notation for a rational number to decimal notation.

EXAMPLE E Find decimal notation for $-\frac{1}{12}$.

Solution

Divide 1 ÷ 12.

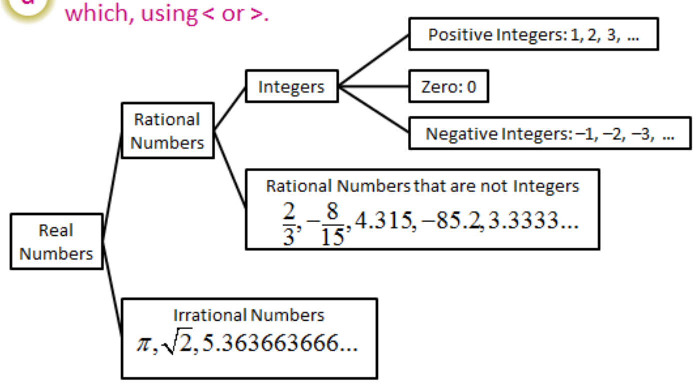
$$12\overline{)1.0000} = 0.0833$$

0
100
96
40
36
40
36
4

Since 4 keeps reappearing as a remainder, the digits repeat and will continue to do so; therefore,

$$-\frac{1}{12} = -0.08333\ldots \text{ or } -\frac{1}{12} = -0.08\overline{3}.$$

(d) Determine which of two real numbers is greater and indicate which, using < or >.



(d) Determine which of two real numbers is greater and indicate which, using < or >.

EXAMPLE F Use either < or > for □ to form a true sentence.

1. − 7 □ 3 2. 8 □ − 3 3. − 21 □ − 9

the one to the right.

The symbol < means "is less than,"

– 4 < 8 is read "– 4 is less than 8."

The symbol > means "is greater than,"

– 6 > – 9 is read "– 6 is greater than – 9."

EXAMPLE F Use either < or > for □ to form a true sentence.

1. – 7 □ 3   2. 8 □ – 3   3. – 21 □ – 9

d Determine which of two real numbers is greater and indicate which, using < or >.

EXAMPLE F Use < or > to form a true sentence.

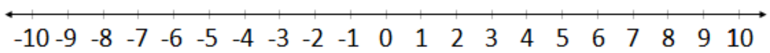
Solution

1. – 7 □ 3

Since –7 is to the **left** of 3, we have –7 < 3.

2. 8 □ – 3

Since 8 is to the **right** of – 3, we have 8 > – 3.



– 7 < 3.   8 > – 3.

(continued)

d Determine which of two real numbers is greater and indicate which, using < or >.

EXAMPLE F Use < or > to form a true sentence.

3. – 21 □ – 9

Since – 21 is to the **left** of – 9, we have –21 < – 9.

– 21 < – 9

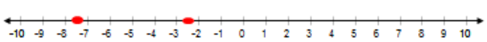
d Determine which of two real numbers is greater and indicate which, using < or >.

EXAMPLE G Use either < or > for □ to form a true sentence.

1. $-7.2 \; \square \; -\frac{5}{2}$ 2. $\frac{5}{8} \; \square \; \frac{8}{13}$

Solution

1. $-7.2 \; \boxed{<} \; -\frac{5}{2}$



d Determine which of two real numbers is greater and indicate which, using < or >.

EXAMPLE G Use either < or > for □ to form a true sentence.

2. $\frac{5}{8} \; \boxed{>} \; \frac{8}{13}$

Convert to decimal notation:

$$\frac{5}{8} = 0.625$$

$$\frac{8}{13} = 0.6154$$

Absolute Value

The absolute value of a number is its distance from zero on a number line. We use the symbol $|x|$ to represent the absolute value of a number x.

To Find the Absolute Value of a Number:

1. If a number is negative, its absolute value is positive.
2. If the number is positive or zero, its absolute value is the same as the number.

e Find the absolute value of a real number.

EXAMPLE H Find the absolute value of each number.

a. | – 5| b. |36|

c. |0| d. | – 52|

Solution

a. | – 5| The distance of – 5 from 0 is 5, so | – 5| = 5.

b. |36| The distance of 36 from 0 is 36, so |36| = 36.

c. |0| The distance of 0 from 0 is 0, so |0| = 0.

d. | – 52| The distance of – 52 from 0 is 52, so | – 52| = 52.

10.2 Addition of Real Numbers

OBJECTIVES

a Add real numbers without using the number line.

b Find the opposite, or additive inverse, of a real number.

Adding Integers

To perform the addition $a + b$, we start at a, and then move according to b.

a) If b is positive, we move to the right.

b) If b is negative, we move to the left.

c) If b is 0, we stay at a.

(a) Add real numbers without using the number line.

EXAMPLE A Add: $3 + (-6)$.

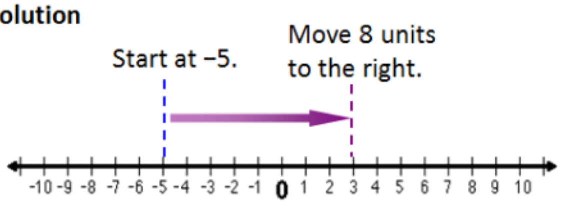
Solution

$3 + (-6) = -3$

(a) Add real numbers without using the number line.

EXAMPLE B Add: $-5 + 8$.

Solution



$-5 + 8 = 3$

Rules for Addition of Real Numbers

1. Positive numbers: Add the same as arithmetic numbers. The answer is positive.
2. Negative numbers: Add absolute values. The answer is negative.

Rules for Addition of Real Numbers

3. A positive and a negative number: Subtract the smaller absolute value from the larger. Then:
   a) If the positive number has the greater absolute value, the answer is positive.
   b) If the negative number has the greater absolute value, the answer is negative.
   c) If the numbers have the same absolute value, the answer is 0.

Rules for Addition of Real Numbers

4. One number is zero: The sum is the other number.

(a) Add real numbers without using the number line.

EXAMPLE C Add: 1. $-5 + (-8) =$ 2. $-9 + (-7) =$

Solution

1. $-5 + (-8) = -13$

Add the absolute values: $5 + 8 = 13$. Make the answer negative.

2. $-9 + (-7) = -16$

(a) Add real numbers without using the number line.

EXAMPLE D Add: 1. $4 + (-6) =$ 2. $12 + (-9) =$ 3. $-8 + 5 =$ 4. $-7 + 5 =$

Solution

1. $4 + (-6) = -2$

Think: The absolute values are 4 and 6. The difference is 2. Since the negative number has the larger absolute value, the answer is *negative*, −2.

2. $12 + (-9) = 3$
3. $-8 + 5 = -3$
4. $-7 + 5 = -2$

a Add real numbers without using the number line.

EXAMPLE E Add: $16 + (-2) + 8 + 15 + (-6) + (-14)$.

Solution

Because of the commutative and associate laws for addition, we can group the positive numbers together and the negative numbers together and add them separately. Then we add the two results.

$16 + (-2) + 8 + 15 + (-6) + (-14)$

$= 16 + 8 + 15 + (-2) + (-6) + (-14)$

$= 39 + (-22)$

$= 17$

Opposites, or Additive Inverses

Two numbers whose sum is 0 are called opposites, or additive inverses, of each other.

b Find the opposite, or additive inverse, of a real number.

EXAMPLE F Find the opposite, or additive inverse, of each number.

1. 52   2. -12   3. 0   4. $-\frac{4}{5}$

Solution

1. 52 The opposite of 52 is -52 because $52 + (-52) = 0$
2. -12 The opposite of -12 is 12 because $-12 + 12 = 0$
3. 0 The opposite of 0 is 0 because $0 + 0 = 0$
4. $-\frac{4}{5}$ The opposite of $-\frac{4}{5}$ is $\frac{4}{5}$ because $-\frac{4}{5} + \frac{4}{5} = 0$

Symbolizing Opposites

The opposite, or additive inverse, of a number a can be named $-a$ (read "the opposite of a," or "the additive inverse of a").

The Opposite of the Opposite

The opposite of the opposite of a number is the number itself. (The additive inverse of the additive inverse of a number is the number itself.) That is, for any number a.

$-(-a) = a$.

b Find the opposite, or additive inverse, of a real number.

EXAMPLE G Evaluate $-x$ and $-(-x)$ when $x = 12$.

Solution

We replace x in each case with 12.

a) If $x = 12$, then $-x = -12 = -12$

b) If $x = 12$, then $-(-x) = -(-12) = 12$

b Find the opposite, or additive inverse, of a real number.

EXAMPLE H Evaluate $-(-x)$ for $x = -7$.

Solution

We replace x with -7.

If $x = -7$, then $-(-x) = -(-(-7)) = -7$

The Sum of Opposites

For any real number a, the opposite, or additive inverse, of a, expressed as $-a$, is such that

$a + (-a) = -a + a = 0$.

b Find the opposite, or additive inverse, of a real number.

EXAMPLE I Change the sign (Find the opposite.)

a) -9 b) 8

Solution

a) -9 $-(-9) = 9$

b) 8 $-(8) = -8$