# Chapter 1

## <u>1.6: Addition of Integers</u>

We can use the number line to add integers. If we have **a + b** then:

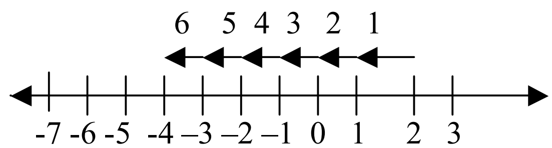
1.) If **b** is positive, we start at **a** and move **b** units right.

2.) If **b** is negative we start at **a** and move **b** units left.

3.) If **b = 0**, stay at **a**.

<u>Example 6</u>: Add 2 and – 6.

Here a = 2 and b = - 6. Hence 2 + (-6) can be found by starting on a number line at 2 and moving 6 units to the left.




Hence, 2 + (-6) = -4.

<u>Example 7</u>: Refer to the number line in example 6 to answer the following;

Add – 3 and – 4 , Add – 5 and 6.

Start at –3 and move 4 units left to get (-3) + (-4) = -7.

Start at –5 and move 6 units right to get -5 + 6 = 1.

When you add two negative numbers together you get a negative number for an answer. When adding negatives, think about adding their absolute values together and then change the sign to make the answer negative.

<u>Example 8:</u> Add – 5 and – 5. So we have –5 + (-5) = -10.

Add – 10 and – 20. So we have – 10 + (- 20) = -30.

When you add a positive and negative number, the sign of the answer will depend on which number has the greatest absolute value.

<u>Example 9</u>: Add 3 and (– 5). So we have 3 + (-5) = - 2. The answer is negative because $|-5| > |3| \rightarrow 5 > 3$.
Add 10 and – 3. So we have 10 + (-3) = 7. The answer is positive because $|10| > |-3| \rightarrow 10 > 3$.

When we want to add several positive and negative numbers up, we use the associative and commutative properties. We group all negative numbers together and all the positive numbers together, then add.

<u>Example 10</u>: Add 5 + (-3) + (-2) + 6 + (-11).

(5 + 6) + [ (-3) + (-2) + (-11)] = 11 + [-16] = -5.

Add 200 + (-200) + 30 + (-20)

[(-200) + 200] + [30 + (-20)] = 0 + 10 = 10.

## <u>1.7: Subtraction of Integers</u>

**<u>Subtraction</u>**, **(a – b)**, is defined as the number, **c**, that when added to **b** gives **a**. Hence**, a – b = c** is the number such that **a = b + c**.

<u>Example 1</u>: 20 – 15 = 5 because 20 = 15 + 5. 10 – 15 = - 5 because 10 = - 5 + 15. You have to think, what number can we add to – 5 to give us 10? We need the absolute value of the number to be greater then $|-5|$, so the answer will be positive. Hence the 15 in the number we seek since $|15| > |-5|$ and –5 + =10.

To subtract, **add** the opposite, or additive inverse of the number being subtracted, we use **a – b = a + (- b)** use **a – b** for shorthand. When subtracting more then two numbers, regroup the numbers into positive and negative numbers, then subtract.

<u>Example 2</u>: Subtract, 100 and 350.

$$100 - 350 = 100 + (-350) = -250.$$

Subtract the following 3 + 5 – 6 – 7 + 8 – 9. Regroup as follows:

$$[3+ 5 + 8] + [- 6 -7 - 9] = [16] + [- 22] = - 5.$$

## <u>1.8: Multiplication of Integers</u>

In **<u>multiplication</u>** of integers, the hard part is determining if the product is positive or negative. Otherwise, multiplication of integers is the same as we did with whole numbers.

<u>RULE 1</u>: When multiplying a *positive* integer with a *negative* integer the answer is **negative**. **(-)(+) = (-)**, AND by the commutative property of multiplication, **(+)(-) = (-)**.

<u>Example 1</u>: (-2)(2) = - 4. Why? (-2)(2) = (-2) + (-2) = -4. Also, (-2)(2) = (2)(-2) = -4.
(-10)(3) = -30. Why? (-10)(3) = (-10) + (-10) + (-10) = -30. Also, (-10)(3) = (3)(-10)=-30.

<u>Rule 2</u>: When multiplying *two negative* integers together, the answer is **positive**, **(-)(-) = (+)**. **NOTE**: Any even number of pairing of negative integers gives a positive product. **(-)(-)(-)(-) = (+)**, and so on.

<u>Example 2</u>: (-2)(-2) = 4.
(-50)(-2)(-1)(-3) = [(-50)(-2)][(-1)(-3)] = (100)(3) = 300.

<u>Rule 3</u>: When multiplying two positive integers, the answer is positive. **(+)(+) = (+)**.

<u>Example 3</u>: (2)(2) = 2 + 2 = 4.

(10)(3) = 10 + 10 + 10 = 30.  (50)(2) = 50 + 50 = 100.

**In General**: If the signs are the same, the answer is positive. If the signs are different, the answer is negative. ALSO, if there is an odd number of negative integers the answer is negative. If there is an even number of negative integers, the answer is positive. When multiplying more then two integers, group negative integers together and positive integers together, simplify, then multiply to get the correct sign for the answer.

Example 4: Evaluate (-2)(3)(-4)(2)(5).

[(3)(2)(5)] • [(-2)(-4)] = [30] • [8] = 240.

Simplify (-2)[(-4)(2)(-3)(-3)](-2). Since there are five negatives, the answer will be negative. (-2)[-72](-2) = (144)(-2) = - 288.

**Powers of Integers**: A *negative* integer raised to an *even power* will be *positive*.

Why? $(-a)^4 = (-a)(-a)(-a)(-a) = (a^2)(a^2) = a^4$.

Example 5: Evaluate $(-2)^4$.

$$(-2)^4 = (-2)(-2)(-2)(-2) = (4)(4) = 16.$$

A *negative* integer raised to an *odd power* will be *negative*. Why?

$(-a)^3 = (-a)(-a)(-a) = (a^2)(-a) = -a^3$.

Example 6: Evaluate $(-3)^3$.

$$(-3)^3 = (-3)(-3)(-3) = (3^2)(-3) = (9)(-3) = -27$$

**NOTE:** **Please NEVER make the following mistake!** $(-a)^2 \neq -a^2$. **In general:**

$$(-a)^{power} \neq -a^{power}$$

Example 7: Show that $(-7)^2 \neq -7^2$.

$$(-7)^2 = (-7)(-7) = 49$$

$$-7^2 = -(7)(7) = -49$$

Hence, $(-7)^2 = 49 \neq -49 = -7^2 \rightarrow (-7)^2 \neq -7^2$

Rule 4: For any integer **a**, **- 1 • a = - a**. That is, if we multiply an integer by – 1, we get the opposite of the integer.

Example 8: What is the opposite of 2?

(-1)(2) = - 2, hence –2 is the opposite of 2.

What is the opposite of – 3?

(-1)(-3) = 3, hence 3 is the opposite of – 3.

**The** multiplication **by zero rule**: For any integer **a**, **a • 0 = 0**. This leads to the **zero product theorem**: If **a** and **b** are numbers, and if **(a)(b) = 0**, then either **a=0** or **b=0**, or both **a** and **b** are zero.

Example 9: Given the fact that (10)(x) = 0, what is the value of x?

By the zero product theorem, since their product is zero, one of the numbers must be zero. Since 10 is not zero, it must be concluded that x = 0.

# 1.8 – 1.9: Division of Integers

When dividing integers, the quotient $\frac{a}{b}$ or $a \div b$, is the number **c**, if it exists, that when multiplied by **b** gives **a**, or **(b)(c) = a**.

Example 1: **Solve** $\frac{24}{12}$. Think: what number when multiplied by 12 gives 24? The answer is 2, because (12)(2) = 24.

**Solve** $\frac{-12}{3}$.  Think: what number when multiplied by 3 gives –12? Since 3 in positive and –12 is negative, the number we multiply by 3 to get –12 must be negative. Hence, -12 = (3)(-4). Here the quotient $\frac{a}{b} = \frac{-12}{3} = -4$.

**Solve** $\frac{-10}{-5}$.  Think: what number when multiplied by –5 gives –10? Since 5 is negative and 10 is negative, the number we seek must be positive. Since –10 = (-5)(2), the quotient we seek is 2.

**NOTE:** If the sign of the numerator and denominator are the **same**, the answer is **positive**! $\frac{(+)}{(+)} = (+)$ *AND* $\frac{(-)}{(-)} = (+)$. However, if the signs of the numerator and denominator are **different**, the answer is **negative**! $\frac{(-)}{(+)} = (-)$ *AND* $\frac{(+)}{(-)} = (-)$.

There is one **RULE** you must always remember: **NEVER DIVIDE BY ZERO!**

For any nonzero number **a**, **a** $\div 0$ or $\frac{a}{0}$, is **UNDEFINED**. Why? $\frac{a}{0}$ implies what number times 0 is **a**? By the zero product theorem, this does not make sense, since **a** $\neq 0$, and any number times 0 is 0. Even if **a** = 0, the form $\frac{0}{0}$ is **UNDEFINED! WE NEVER DIVIDE BY ZERO!**

Example 2: **Solve** $\frac{5}{0}$. This is not solvable since the denominator is 0!

**Solve** for x in $\frac{x}{0}$. Again this is not solvable since the denominator is 0!

However, if you have for any <u>nonzero number **b**</u>, the quotient $\frac{0}{b}$ this is equal to 0, because what number times **b** equals 0? The only answer to this question is 0. Hence, the only way to make a **fraction 0 is for the numerator to be zero**. NOTICE: zero in the denominator makes the fraction undefined, zero in the numerator makes the fraction zero!

<u>Example 3</u>: Solve $\frac{0}{110}$. Think: what number times 11 equals 0? The only answer in zero. Hence the quotient $\frac{0}{110}$ = 0, since (0)(110) = 0.

The **<u>rules for order of operations</u>** for integers are the same as before. However now you must be careful with signs.

I.) Do all calculations in brackets first. Simplify fraction bars.

II.) Evaluate all exponential expressions.

III.) Do all multiplication/division in order from left to right.

IV.) Do all addition/subtraction in order from left to right.

<u>NOTE</u>: Fraction bars separate the calculations in the numerator from calculations in the denominator.

Example 4: Simplify the expression $9-\frac{|3-6|^3}{(-3)(-3)}+26\bullet 2$.

$$9-\frac{|3-6|^3}{(-3)(-3)}+26\bullet 2=9-\frac{|-3|^3}{9}+52=9-\frac{3^3}{3}+52=9-\frac{27}{9}+52=9-3+52=6+52=58$$

**Algebraic expressions** are expressions such as; x + 2 = 5, $\frac{a}{b}$, or 2y = 20. In all of these expressions we see letters called **variables**. Variables are letters that are place- holders for numbers. When a variable holds the place for a particular number, we call that variable a **constant**. Hence, an algebraic expression consists of variables, constants and operation signs. If we replace the variable with a number this is called **substituting** for the variable. This is then said to be **evaluating** the algebraic expression.

Example 1: Evaluate the algebraic expression **x – y**, for **x** = 2 and **y** = 10.
Substituting 2 for **x** and 10 for **y** in **x – y**, we get 2 – 10 = -8. Hence, -8 is the value of the algebraic expression.